

Functions, relations and function notation

Mathematics Advanced · Year 11 · Starting

Use the worked examples and teaching notes to guide the lesson. Each practice answer and its full working appear beside the question.

Take the time you need. Complete each phase and revisit any step you cannot yet explain.

Get ready

Learning goal

I can read a function, work out an output, find a simple input and check what the answer means.

- I can identify input, rule and output in a function.
- I can substitute a positive or negative input and calculate in order.
- I can find an input for a stated linear output using equal operations.
- I can interpret and check a function model in its stated context.

The idea in context

A rule connects inputs to outputs. A reliable solution starts by deciding which quantity the question gives you.

Before you begin

Signed numbers, squares and order of operations

Use this bridge whenever a readiness answer or its reasoning is uncertain. Complete the worked model, fill the gaps, then try the fresh check.

factor, term, square, inverse operation: Factors multiply; terms add. A square has two equal factors. An inverse operation undoes another operation.

Watch the prerequisite 1

Work out $2 \times (-3) + 5$ and $(-3)^2$.

Step 1

Before: $2 \times (-3)$

Action: Add two groups of negative three.

After: $(-3) + (-3) = -6$.

Why: Two steps of three to the left of zero end at negative six.

Start 0; subtract 3: -3 ; subtract 3: -6

$$-6 + 5 = -1$$

$$(-3)^2 = (-3) \times (-3) = 9$$

Track each operation

Start 0; subtract 3: -3 ; subtract 3: -6 $-6 + 5 = -1$ $(-3)^2 = (-3) \times (-3) = 9$

Step 2

Before: $-6 + 5$

Action: Move five units right from negative six.

After: $-5, -4, -3, -2, -1$.

Why: Adding a positive number moves right. The result is negative one.

Step 3

Before: $(-3)^2$

Action: Write the two equal factors.

After: $(-3) \times (-3) = 9$.

Why: A square multiplies the whole bracketed number by itself. Two negative factors give a positive product.

Step 4

Before: -3^2

Action: Compare without the brackets.

After: $-(3 \times 3) = -9$.

Why: An exponent acts before an unbracketed leading minus. These expressions are different.

Step 5

Before: Multiply negative two by a decreasing integer.

Action: Keep the same change between neighbouring products.

After: $(-2) \times 2 = -4$; $(-2) \times 1 = -2$; $(-2) \times 0 = 0$; $(-2) \times (-1) = 2$; $(-2) \times (-2) = 4$.

Why: As the second factor decreases by one, the product increases by two. This preserves the multiplication pattern across zero.

$$(-2) \times 2 = -4$$

$$(-2) \times 1 = -2$$

$$(-2) \times 0 = 0$$

$$(-2) \times (-1) = 2$$

$$(-2) \times (-2) = 4$$

Extend the same multiplication pattern

$$(-2) \times 2 = -4 \quad (-2) \times 1 = -2 \quad (-2) \times 0 = 0 \quad (-2) \times (-1) = 2 \quad (-2) \times (-2) = 4$$

Step 6

Before: $4 - (-6)$

Action: Undo adding negative six.

After: $4 - (-6) = 4 + 6 = 10$.

Why: Subtracting a number adds its opposite. Check the inverse: $10 + (-6) = 4$.

Step 7

Before: $6/(-2)$

Action: Find the missing factor.

After: $(-2) \times ? = 6$; $(-2) \times (-3) = 6$; therefore $6/(-2) = -3$.

Why: Division asks which factor produces the dividend. A nonzero negative divisor is valid.

$2 \times (-3) + 5 = -6 + 5 = -1$; $(-3)^2 = 9$; $-3^2 = -9$.

Complete the missing step 1

Complete: $(-4)^2 = (-4) \times \underline{\quad} = \underline{\quad}$.

Hint 1: Identify what is given.

Hint 2: Follow the matching bridge model.

Hint 3: Perform the displayed operation, then check the original expression.

The equal factors are -4 and -4 . Their product is 16 .

Try a fresh question 1

Evaluate $2 \times (-4) + 3$.

Hint 1: Identify the operation first.

Hint 2: Use brackets and equal operations where needed.

Hint 3: Check each arithmetic line separately.

Multiply first: $2 \times (-4) = -8$. Add three: -7 , -6 , -5 . Result -5 .

Keep an equation balanced

Use this bridge whenever a readiness answer or its reasoning is uncertain. Complete the worked model, fill the gaps, then try the fresh check.

equation, equals, nonzero: An equation says two amounts are equal. Apply the same valid operation to both sides. Nonzero means not zero; division by zero is undefined.

Watch the prerequisite 1

Solve $3x - 10 = 8$.

Step 1

Before: $3x - 10 = 8$

Action: Add ten to both sides.

After: $3x - 10 + 10 = 8 + 10$; $3x = 18$.

Why: The opposite terms -10 and $+10$ sum to zero.

$$\begin{array}{rcl} 3x - 10 & = & 8 \\ +10 & & +10 \\ \hline \end{array}$$

$$3x \quad \quad \quad \cancel{-10} \quad \quad \quad \cancel{+10} \quad \quad = \quad 18$$

Opposite terms sum to zero.

$$3x \quad \quad \quad = \quad \quad \quad 18$$

Apply $+10$ to both sides

$3x - 10$ equals 8. Apply $+10$ to each side. Result: $3x$ equals 18. The marked opposite terms sum to zero.

Step 2

Before: $3x = 18$

Action: Divide both sides by three.

After: $(3 \times x)/3 = 18/3$; $1 \times x = 6$; $x = 6$.

Why: The matching nonzero factors give $3/3 = 1$, not zero.

$$(3 \times x)/3 = 18/3$$

$$(3/3) \times x = 6$$

$$1 \times x = 6$$

$$x = 6$$

Factors reduce to one

$$(3 \times x)/3 = 18/3 \quad (3/3) \times x = 6 \quad 1 \times x = 6 \quad x = 6$$

Step 3

Before: $x = 6$

Action: Substitute into the original equation.

After: $3 \times 6 - 10 = 18 - 10 = 8$.

Why: The original sides agree.

$x = 6$.

Complete the missing step 1

Complete: $2x - 6 = 4$. Add ___ to both sides; $2x = \underline{\hspace{1cm}}$; $x = \underline{\hspace{1cm}}$.

Hint 1: Identify what is given.

Hint 2: Follow the matching bridge model.

Hint 3: Perform the displayed operation, then check the original expression.

$$2x - 6 = 4.$$

Add 6 on both sides:

$$2x - 6 + 6 = 4 + 6.$$

$$2x + 0 = 10; 2x = 10.$$

Divide both sides by 2:

$$(2 \times x)/(2) = 10/(2).$$

$$(2/2) \times x = 5; 1 \times x = 5.$$

$$x = 5.$$

$$\text{Check: } 2 \times (5) - 6 = 10 - 6 = 4.$$

Try a fresh question 1

$$\text{Solve } 4x + 1 = 13.$$

Hint 1: Identify the operation first.

Hint 2: Use brackets and equal operations where needed.

Hint 3: Check each arithmetic line separately.

$$4x + 1 = 13.$$

Subtract 1 on both sides:

$$4x + 1 - 1 = 13 - 1.$$

$$4x + 0 = 12; 4x = 12.$$

Divide both sides by 4:

$$(4 \times x)/(4) = 12/(4).$$

$$(4/4) \times x = 3; 1 \times x = 3.$$

$$x = 3.$$

$$\text{Check: } 4 \times (3) + 1 = 12 + 1 = 13.$$

A half as an input

Use this bridge before substituting a fractional input.

numerator and denominator: In $1/2$, the denominator 2 divides a whole into two equal parts. The numerator 1 counts one of those parts.

Watch the prerequisite 1

Find six halves, then subtract one.

Step 1

Before: $6 \times (1/2)$

Action: Write the six equal parts.

After: $1/2 + 1/2 + 1/2 + 1/2 + 1/2 + 1/2$.

Why: A half is one of two equal parts of a whole.

$$1/2 + 1/2 = 1$$

$$1/2 + 1/2 = 1$$

$$1/2 + 1/2 = 1$$

$$6 \text{ halves} = 3 \text{ wholes}$$

Pair the halves

$$1/2 + 1/2 = 1 \quad 1/2 + 1/2 = 1 \quad 1/2 + 1/2 = 1 \quad 6 \text{ halves} = 3 \text{ wholes}$$

Step 2

Before: $6/2$

Action: Group the halves in pairs.

After: $6/2 = 3$.

Why: Two halves make each whole; six halves make three wholes.

Step 3

Before: $3 - 1$

Action: Subtract one whole.

After: $3 - 1 = 2$.

Why: Check by adding the removed whole: $2 + 1 = 3$.

$$6 \times (1/2) - 1 = 6/2 - 1 = 3 - 1 = 2.$$

Complete the missing step 1

Complete: $8 \times (1/2) = 8/2 = \underline{\hspace{1cm}}$ wholes.

Hint 1: Each whole needs two halves.

Hint 2: Group eight parts in pairs.

Hint 3: Count the pairs: (1, 2), (3, 4), (5, 6), (7, 8).

Eight halves form four pairs, so $8/2 = 4$.

Try a fresh question 1

Find $10 \times (1/2) - 2$.

Hint 1: Calculate the halves before subtracting.

Hint 2: Divide ten by two.

Hint 3: Five wholes minus two wholes leaves the answer.

Ten halves form five pairs: $10/2 = 5$. Subtract two: $5 - 2 = 3$. Check $3 + 2 = 5$.

Starting-point question 1

Evaluate $3 \times (-2)$.

Answer and working

-6: three groups of negative two.

Starting-point question 2

Compare $(-2)^2$ and -2^2 .

Answer and working

$(-2)^2 = (-2)(-2) = 4$. Without brackets, $-2^2 = -(2 \times 2) = -4$.

Starting-point question 3

Which operation undoes adding seven?

Answer and working

Subtract seven; apply it to both sides of an equation.

Starting-point question 4

Is adding three only to the left side of $2x=6$ a valid way to keep equality?

Answer and working

No. Apply the same operation to both sides to preserve the solutions: $2x+3=6+3$.

Starting-point question 5

How many halves make one whole?

Answer and working

Two equal halves.

Starting-point question 6

How many wholes are four halves?

Answer and working

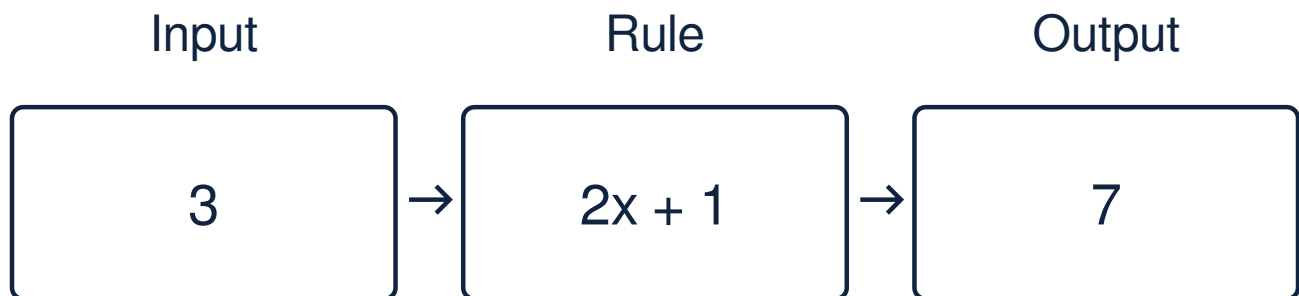
Pair them: two halves and two halves make two wholes.

Watch — I do

Read the function notation

A function assigns exactly one output to each permitted input. Its name is f ; the bracket holds the input. Different inputs may share one output. One input cannot have two outputs under the same rule.

$f(3)$ means the output when $x = 3$.



Input → rule → output

Input 3; rule $2x + 1$; output 7.

When the input is known

Write the rule. Replace every x by the bracketed input. Calculate powers, then multiplication or division, then addition or subtraction. Write the result using function notation.

$$f(x) = x^2 + 2x; f(-2) = (-2)^2 + 2(-2)$$

0

$$(-2)^2 + 2(-2)4 + (-4)0$$
$$f(x) = 3x - 10; f(x) = 8 \Rightarrow 3x - 10 = 8$$

$$3x = 18$$

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Page 11 of 246

If the question gives $f(4)$, the input is four: substitute. If it gives $f(x)=4$, the output is four: solve the equation for x . A table lookup is enough only when the needed input is listed.

Known input $\rightarrow$ substitute; known output $\rightarrow$ solve.

$f(4)$: input is 4

$f(x) = 4$: output is 4

Read the question before calculating.

Input or output?

$f(4)$: input is 4 $f(x) = 4$: output is 4 Read the question before calculating.

Read a model in context

Define the variable, its unit and permitted values. A rule for whole tickets has whole-number inputs. A temperature rule may accept negative values. An algebraic answer can be impossible in the situation.

Cost $C(n) = 4n + 7$ dollars; n is a nonnegative whole number.



Input $\rightarrow$ rule $\rightarrow$ output

Input 2 tickets; rule $4n + 7$; output \$15.

Check rather than memorise a shortcut

$f(x)$ is not f times x . $f(-3)$ is not necessarily $-f(3)$. A pair of outputs can be equal for different inputs, such as $(-2)^2$ and 2^2 . Division cancels common factors of a whole expression, not one term of a sum.

$(x + 6)/3 = x/3 + 2$, not $x + 2$.

$$(x + 6)/3$$

$$= x/3 + 6/3$$

$$= x/3 + 2$$

Check $x = 3$: $9/3 = 3$, not 5.

Divide every term

$$(x + 6)/3 = x/3 + 6/3 = x/3 + 2 \text{ Check } x = 3: 9/3 = 3, \text{ not } 5.$$

Watch — worked models

Follow the model

For $f(x) = 2x + 1$, find $f(3)$.

Step 1

Before: $f(x) = 2x + 1$; find $f(3)$.

Action: Read the input inside the brackets.

After: The input is 3.

Why: f names the rule. $f(x)$ is its output; it is not f multiplied by x .

Step 2

Before: $2x + 1$

Action: Replace every x with the input in brackets.

After: $f(3) = 2(3) + 1$

Why: Brackets keep a negative input or a whole expression together.

$$f(x) = 2x + 1$$

$$\text{input } x = 3$$

$$f(3) = 2(3) + 1$$

Keep the input together

$$f(x) = 2x + 1 \quad \text{input } x = 3 \quad f(3) = 2(3) + 1$$

Step 3

Before: $2(3) + 1$

Action: Carry out one operation.

After: $6 + 1$

Why: Multiply two by three first.

Step 4

Before: $6 + 1$

Action: Carry out one operation.

After: 7

Why: Add the remaining one.

Step 5

Before: $f(3) = 7$.

Action: Check against the rule.

After: Start with three; doubling gives six; adding one gives seven.

Why: Recalculate in the original rule, including each sign.

$f(3) = 7$. Start with three; doubling gives six; adding one gives seven.

Follow the model

For $f(x) = x^2 + 2x$, find $f(-2)$.

Step 1

Before: $f(x) = x^2 + 2x$; find $f(-2)$.

Action: Read the input inside the brackets.

After: The input is -2 .

Why: f names the rule. $f(x)$ is its output; it is not f multiplied by x .

Step 2

Before: $x^2 + 2x$

Action: Replace every x with the input in brackets.

After: $f(-2) = (-2)^2 + 2(-2)$

Why: Brackets keep a negative input or a whole expression together.

$$f(x) = x^2 + 2x$$

$$\text{input } x = -2$$

$$f(-2) = (-2)^2 + 2(-2)$$

Keep the input together

$$f(x) = x^2 + 2x \text{ input } x = -2 \quad f(-2) = (-2)^2 + 2(-2)$$

Step 3

Before: $(-2)^2 + 2(-2)$

Action: Carry out one operation.

After: $4 + 2(-2)$

Why: Square the bracketed input: $(-2) \times (-2) = 4$.

Step 4

Before: $4 + 2(-2)$

Action: Carry out one operation.

After: $4 + (-4)$

Why: Multiply $2 \times (-2) = -4$.

Step 5

Before: $4 + (-4)$

Action: Carry out one operation.

Why: Opposite terms sum to zero.

Step 6

Before: $f(-2) = 0$.

Action: Check against the rule.

After: The two terms are four and negative four, which balance to zero.

Why: Recalculate in the original rule, including each sign.

$f(-2) = 0$. The two terms are four and negative four, which balance to zero.

Follow the model

$f(x) = 3x - 10$. Find x when $f(x) = 8$.

Step 1

Before: $f(x) = 8$

Action: Identify the output, not the input.

After: The unknown is x . Set $3x - 10 = 8$.

Why: The rule produces the stated output.

Step 2

Before: $3x - 10 = 8$

Action: Add 10 to both sides.

After: $3x - 10 + 10 = 8 + 10$; $3x + 0 = 18$; $3x = 18$.

Why: The constant and its opposite sum to zero.

$$\begin{array}{rcl} 3x - 10 & = & 8 \\ +10 & & +10 \\ \hline 3x & \overset{=}{\cancel{-10}} \quad \overset{+}{\cancel{+10}} & = 18 \end{array}$$

Opposite terms sum to zero.

$$3x = 18$$

Apply +10 to both sides

$3x - 10$ equals 8. Apply +10 to each side. Result: $3x$ equals 18. The marked opposite terms sum to zero.

Step 3

Before: $3x = 18$

Action: Divide both sides by 3.

After: $(3 \times x)/(3) = 18/(3)$; $(3/3) \times x = 6$; $1 \times x = 6$; $x = 6$.

Why: The matching nonzero coefficient factors reduce to one.

Divide both sides by 3.

$$\frac{\cancel{3} \times x}{\cancel{3}} = \frac{18}{3}$$

$3/3 = 1$ because 3 is not zero.

$1 \times x = 6$; $x = 6$.

Matching nonzero factors reduce to one

Divide 3 times x equals 18 on both sides by 3. The matching factors 3 reduce to one, leaving x equals 6.

Step 4

Before: $x = 6$

Action: Substitute in the original rule.

After: $3 \times (6) - 10 = 18 - 10 = 8$.

Why: The recovered output matches the question.

$x = 6$.

Follow the model

$f(x) = -2x + 5$. Find x when $f(x) = 11$.

Step 1

Before: $f(x) = 11$

Action: Identify the output, not the input.

After: The unknown is x . Set $-2x + 5 = 11$.

Why: The rule produces the stated output.

Step 2

Before: $-2x + 5 = 11$

Action: Subtract 5 from both sides.

After: $-2x + 5 - 5 = 11 - 5$; $-2x + 0 = 6$; $-2x = 6$.

Why: The constant and its opposite sum to zero.

$$\begin{array}{rcl}
 -2x + 5 & = & 11 \\
 -5 & & -5 \\
 \hline
 -2x & \quad \quad \quad \cancel{+5} \quad \quad \quad \cancel{-5} & = 6
 \end{array}$$

Opposite terms sum to zero.

$$-2x = 6$$

Apply -5 to both sides

$-2x + 5$ equals 11. Apply -5 to each side. Result: $-2x$ equals 6. The marked opposite terms sum to zero.

Step 3

Before: $-2x = 6$

Action: Divide both sides by -2 .

After: $(-2 \times x)/(-2) = 6/(-2)$; $(-2/-2) \times x = -3$; $1 \times x = -3$; $x = -3$.

Why: The matching nonzero coefficient factors reduce to one.

Divide both sides by -2 .

$$\frac{\cancel{-2} \times x}{\cancel{-2}} = \frac{6}{-2}$$

$-2/-2 = 1$ because -2 is not zero.

$1 \times x = -3$; $x = -3$.

Matching nonzero factors reduce to one

Divide -2 times x equals 6 on both sides by -2 . The matching factors -2 reduce to one, leaving x equals -3 .

Step 4

Before: $x = -3$

Action: Substitute in the original rule.

After: $-2 \times (-3) + 5 = 6 + 5 = 11$.

Why: The recovered output matches the question.

$x = -3$.

Follow the model

A fictional ticket order costs $C(n)=4n+7$ dollars. n is a nonnegative whole number. What does an order of 3 tickets cost?

Step 1

Before: $n = 3$ tickets

Action: Identify the unknown and the units.

After: Find $C(3)$, measured in dollars.

Why: The count of tickets is the input; cost is the output.

Step 2

Before: $C(n)=4n+7$

Action: Replace n with three.

After: $C(3)=4(3)+7$.

Why: The seven-dollar charge is applied once, not once per ticket.



Input → rule → output

Input 3 tickets; rule $4n + 7$; output \$19.

Step 3

Before: $4(3)+7$

Action: Multiply, then add.

After: $4 \times 3 = 12$; $12 + 7 = 19$.

Why: Three four-dollar tickets cost twelve, plus the fixed seven.

Step 4

Before: $C(3)=19$

Action: Check a repeated-addition total.

After: $4+4+4+7=19$ dollars.

Why: The independent calculation agrees; answer the question in dollars.

The order costs \$19.

Follow the model

Under $C(n)=4n+7$, can an order cost \$17? Only whole tickets can be bought.

Step 1

Before: $C(n)=17$

Action: Set the rule equal to the cost.

After: $4n+7=17$.

Why: The cost is an output, so solve for its input.

Step 2

Before: $4n+7=17$

Action: Subtract seven from both sides.

After: $4n+7-7=17-7$; $4n=10$.

Why: The opposite constants sum to zero.

$$\begin{array}{rclcl}
 4n + 7 & = & 17 \\
 -7 & & -7 \\
 \hline
 4n & \overset{+7}{\cancel{-7}} & \overset{-7}{\cancel{+7}} & = & 10
 \end{array}$$

Opposite terms sum to zero.

$$4n = 10$$

Apply -7 to both sides

$4n + 7$ equals 17. Apply -7 to each side. Result: $4n$ equals 10. The marked opposite terms sum to zero.

Step 3

Before: $4n=10$

Action: Divide both sides by four.

After: $(4 \times n)/4 = 10/4$; $(4/4) \times n = 2.5$; $1 \times n = 2.5$; $n = 2.5$.

Why: Four fourths make one. Check: $4 \times 2.5 + 7 = 10 + 7 = 17$.

Divide both sides by 4.

$$\frac{\cancel{4} \times n}{\cancel{4}} = \frac{10}{4}$$

$4/4 = 1$ because 4 is not zero.

$1 \times n = 2.5$; $n = 2.5$.

Matching nonzero factors reduce to one

Divide 4 times n equals 10 on both sides by 4. The matching factors 4 reduce to one, leaving n equals 2.5.

Step 4

Before: $n=2.5$ tickets

Action: Check the permitted inputs.

After: 2.5 is not a whole number. An exact \$17 order is impossible.

Why: $C(2) = 4(2) + 7 = 8 + 7 = 15$ dollars. $C(3) = 4(3) + 7 = 12 + 7 = 19$ dollars. Rounding changes the output.

No exact \$17 order is allowed. The equation gives 2.5 tickets, which violates the input condition.

Follow the model

Does the table $(1, 4)$, $(2, 5)$, $(3, 5)$ describe a function? Each pair is (input, output).

Step 1

Before: Pairs $(1, 4)$, $(2, 5)$, $(3, 5)$

Action: Group rows by their input.

After: Input $1 \rightarrow 4$; input $2 \rightarrow 5$; input $3 \rightarrow 5$.

Why: Check outputs for the same input, not inputs for the same output.

Input 1: output 4

Input 2: output 5

Input 3: output 5

One output for each input

Input 1: output 4 Input 2: output 5 Input 3: output 5

Step 2

Before: Each input appears with one output.

Action: Apply the definition.

After: This is a function on inputs $\{1, 2, 3\}$.

Why: Inputs 2 and 3 sharing output 5 is allowed.

Yes, each listed input has exactly one output.

Follow the model

Does the table $(1, 4)$, $(1, 6)$, $(2, 5)$ describe a function?

Step 1

Before: Pairs $(1, 4)$, $(1, 6)$, $(2, 5)$

Action: Group the repeated input.

After: Input $1 \rightarrow 4$ and input $1 \rightarrow 6$.

Why: The same input must not demand different outputs.

Input 1: output 4

Input 1: output 6

Input 2: output 5

One input, two different outputs

Input 1: output 4 Input 1: output 6 Input 2: output 5

Step 2

Before: One input has two outputs.

Action: Apply the definition.

After: This relation is not a function of the input.

Why: The row for input 2 cannot repair the conflict at input 1.

No: input 1 is assigned both 4 and 6.

Follow the model

Only $f(0)=1$ and $f(1)=2$ are known. Must $f(2)$ equal three?

Step 1

Before: Two input-output pairs, no rule.

Action: Separate the facts from the proposed pattern.

After: Known: $0 \rightarrow 1$ and $1 \rightarrow 2$. Unknown: the rule and output for input 2.

Why: A table lookup cannot supply a missing row.

Step 2

Before: Try $f(x)=x+1$.

Action: Check both known inputs.

After: $f(0)=0+1=1$; $f(1)=1+1=2$. Then $f(2)=2+1=3$.

Why: This rule fits, but one fitting rule need not be the only rule.

$$f(x) = x + 1$$

$$0 + 1 = 1$$

$$1 + 1 = 2$$

$$2 + 1 = 3$$

One possible rule

$$f(x) = x + 1 \quad 0 + 1 = 1 \quad 1 + 1 = 2 \quad 2 + 1 = 3$$

Step 3

Before: Try $g(x)=x^2+1$.

Action: Check the same inputs with another rule.

After: $g(0)=0^2+1=0+1=1$; $g(1)=1^2+1=1+1=2$. But $g(2)=2^2+1=4+1=5$.

Why: Both rules fit the facts but give different missing outputs.

$$g(x) = x^2 + 1$$

$$0^2 + 1 = 1$$

$$1^2 + 1 = 2$$

$$2^2 + 1 = 5$$

A second possible rule

$$g(x) = x^2 + 1 \quad 0^2 + 1 = 1 \quad 1^2 + 1 = 2 \quad 2^2 + 1 = 5$$

Step 4

Before: Possible outputs include 3 and 5.

Action: State what the evidence supports.

After: The output is not determined. Ask for the rule or more conditions.

Why: Do not turn a plausible pattern into a claim of certainty.

No. The two given pairs alone do not determine $f(2)$.

Together — We do

Follow the guided example

For $f(x)=2x+3$, complete $f(5)$.

Step 1

Before: $f(x)=2x+3$; input 5

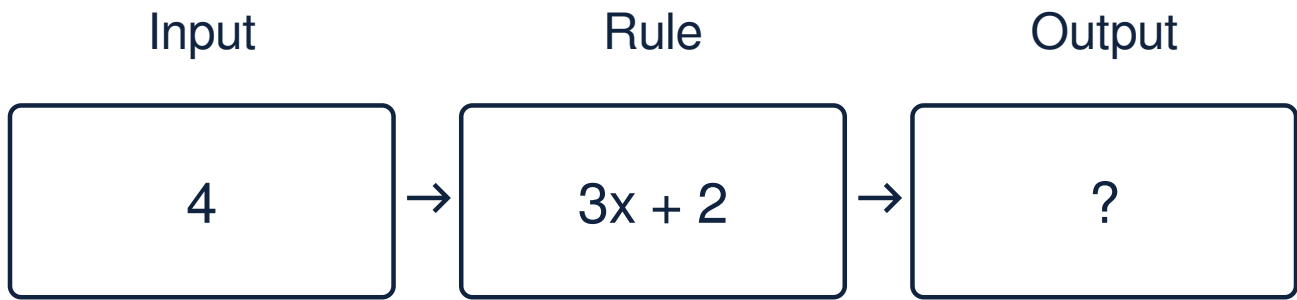
Action: Replace each x with five.

After: $f(5)=2(\underline{\quad})+3=\underline{\quad}+3=\underline{\quad}$.

Why: Multiply first, then add the fixed term.

$f(5)=2(5)+3=10+3=13$. Check $5+5+3=13$.

Question 1



Input → rule → output

Input 4; rule $3x + 2$; output ?.

For $f(x)=3x+2$, complete $f(4)=3(\underline{\quad})+2=\underline{\quad}+2=\underline{\quad}$.

Answer and working**Step 1**

Before: $f(4)$

Action: Read the supplied input.

Operation: $f(4) \rightarrow x=4$

After: $x=4$

Why: The number inside the brackets is the input; the required result is the output.

$$f(4)$$

$$x=4$$

Read the supplied input.

$f(4)$; $x=4$

Step 2

Before: $f(x)=3x+2$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=3x+2 \rightarrow f(4)=3(4) + 2$

After: $f(4)=3(4) + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=3x+2$$

$$f(4)=3(4) + 2$$

Replace each occurrence of the input variable.

$f(x)=3x+2$; $f(4)=3(4) + 2$

Step 3

Before: $(3) \times (4)$

Action: Multiply the coefficient and the input.

Operation: $(3) \times (4) \rightarrow 12$

After: 12

Why: Complete multiplication before addition or subtraction.

$$(3) \times (4)$$

$$12$$

Multiply the coefficient and the input.

$(3) \times (4)$; 12

Step 4

Before: $(12)+(2)$

Action: Apply the constant term.

Operation: $(12)+(2) \rightarrow 14$

After: 14

Why: The constant is added once after the variable terms have been evaluated.

$$(12)+(2)$$

$$14$$

Apply the constant term.

$(12)+(2)$; 14

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(4)=14$

After: $f(4)=14$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(4)=14$$

Use the original function name.

Label the computed output; $f(4)=14$

Step 6

Before: Fill the original gaps

Action: Use input, product and final output in order.

Operation: Fill the original gaps $\rightarrow f(4)=3(4)+2=12+2=14$; $4+4+4+2=14$

After: $f(4)=3(4)+2=12+2=14$; $4+4+4+2=14$

Why: The repeated-addition check agrees with the coefficient product.

Fill the original gaps

$$f(4)=3(4)+2=12+2=14; 4+4+4+2=14$$

Use input, product and final output in order.

Fill the original gaps; $f(4)=3(4)+2=12+2=14$; $4+4+4+2=14$

1. Read the supplied input. $f(4) \rightarrow x=4$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $f(x)=3x+2 \rightarrow f(4)=3(4) + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(3) \times (4) \rightarrow 12$ Complete multiplication before addition or subtraction.
4. Apply the constant term. $(12)+(2) \rightarrow 14$ The constant is added once after the variable terms have been evaluated.
5. Use the original function name. Label the computed output $\rightarrow f(4)=14$ The requested input has now passed through every operation in the rule.
6. Use input, product and final output in order. Fill the original gaps $\rightarrow f(4)=3(4)+2=12+2=14$; $4+4+4+2=14$ The repeated-addition check agrees with the coefficient product.

Answer: $f(4)=14$

The input is 4. $f(4)=3(4)+2$.

Multiply $3 \times 4=12$.

Add 2:14.

Check repeated addition $4+4+4+2=14$.

What to look for

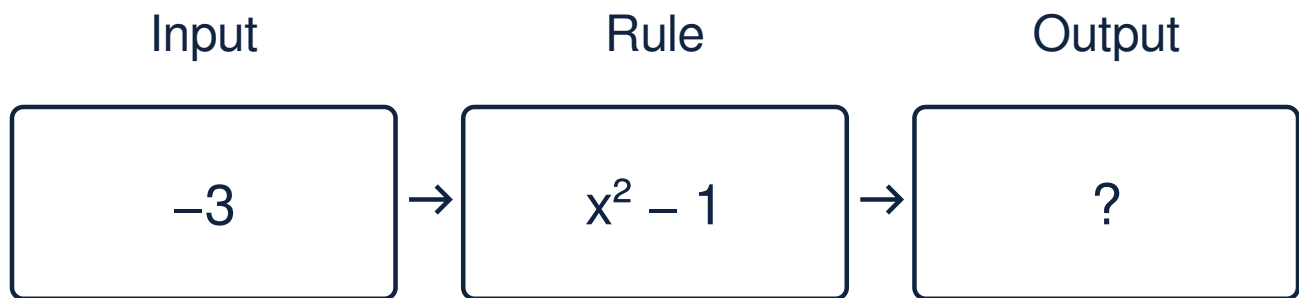
- Selects the correct method.
- Shows intermediate calculations.
- Checks notation and allowed inputs.

Hints to offer

The bracket gives input four.

Replace x in $3x + 2$ by (4) .

$3 \times 4 = 12$. Finish $12 + 2$, then check.

Question 2

Input $\rightarrow$ rule $\rightarrow$ output

Input -3 ; rule $x^2 - 1$; output $?$.

For $g(x)=x^2-1$, complete $g(-3)=(\underline{\hspace{1cm}})^2-1=\underline{\hspace{1cm}}-1=\underline{\hspace{1cm}}$.

Answer and working

Step 1

Before: $g(-3)$

Action: Read the supplied input.

Operation: $g(-3) \rightarrow x=-3$

After: $x=-3$

Why: The number inside the brackets is the input; the required result is the output.

$$g(-3)$$

$$x=-3$$

Read the supplied input.

$g(-3); x=-3$

Step 2

Before: $g(x)=x^2-1$

Action: Replace each occurrence of the input variable.

Operation: $g(x)=x^2-1 \rightarrow g(-3)=(-3)^2 + -1$

After: $g(-3)=(-3)^2 + -1$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$g(x)=x^2-1$$

$$g(-3)=(-3)^2 + -1$$

Replace each occurrence of the input variable.

$g(x)=x^2-1; g(-3)=(-3)^2 + -1$

Step 3

Before: $(-3)^2$

Action: Write the square as a product.

Operation: $(-3)^2 \rightarrow (-3) \times (-3)$

After: $(-3) \times (-3)$

Why: Both factors contain the whole input, including its sign.

$$(-3)^2$$

$$(-3) \times (-3)$$

Write the square as a product.

$(-3)^2$; $(-3) \times (-3)$

Step 4

Before: $(-3) \times (-3)$

Action: Multiply the two identical inputs.

Operation: $(-3) \times (-3) \rightarrow 9$

After: 9

Why: Two negative factors give a positive product.

$$(-3) \times (-3)$$

$$9$$

Multiply the two identical inputs.

$(-3) \times (-3)$; 9

Step 5

Before: $(9)+(-1)$

Action: Apply the constant term.

Operation: $(9)+(-1) \rightarrow 8$

After: 8

Why: Adding a negative constant is the same as subtracting its positive magnitude.

$$(9)+(-1)$$

$$8$$

Apply the constant term.

$(9)+(-1)$; 8

Step 6

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow g(-3)=8$

After: $g(-3)=8$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$g(-3)=8$$

Use the original function name.

Label the computed output; $g(-3)=8$

Step 7

Before: Fill the original gaps

Action: Keep the negative input in its bracket.

Operation: Fill the original gaps $\rightarrow g(-3)=(-3)^2-1=9-1=8$

After: $g(-3)=(-3)^2-1=9-1=8$

Why: Without brackets, -3^2 means the negative of 3^2 , a different expression.

Fill the original gaps

$$g(-3)=(-3)^2-1=9-1=8$$

Keep the negative input in its bracket.

Fill the original gaps; $g(-3)=(-3)^2-1=9-1=8$

1. Read the supplied input. $g(-3) \rightarrow x=-3$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $g(x)=x^2-1 \rightarrow g(-3)=(-3)^2 + -1$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Write the square as a product. $(-3)^2 \rightarrow (-3) \times (-3)$ Both factors contain the whole input, including its sign.
4. Multiply the two identical inputs. $(-3) \times (-3) \rightarrow 9$ Two negative factors give a positive product.
5. Apply the constant term. $(9)+(-1) \rightarrow 8$ Adding a negative constant is the same as subtracting its positive magnitude.
6. Use the original function name. Label the computed output $\rightarrow g(-3)=8$ The requested input has now passed through every operation in the rule.
7. Keep the negative input in its bracket. Fill the original gaps $\rightarrow g(-3)=(-3)^2-1=9-1=8$ Without brackets, -3^2 means the negative of 3^2 , a different expression.

Answer: $g(-3)=8$

Put -3 in brackets: $(-3)^2-1$.

Multiply $(-3) \times (-3)=9$.

Subtract 1: 8 .

Check $9-1=8$; using -3^2 would be a different expression.

What to look for

- Selects the correct method.
- Shows intermediate calculations.
- Checks notation and allowed inputs.

Hints to offer

The input is negative three.

Write $(-3)^2 - 1$, keeping the brackets.

$(-3) \times (-3) = 9$. Finish $9 - 1$.

Question 3

For $h(x)=2x-5$, find x when $h(x)=9$. Fill the same added quantity under both sides.

Answer and working**Step 1**

Before: $h(x)=9$

Action: The output is known; the input is unknown.

Operation: $h(x)=9 \rightarrow 2x-5=9$

After: $2x-5=9$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$h(x)=9$$

$$2x-5=9$$

The output is known; the input is unknown.

$h(x)=9$; $2x-5=9$

Step 2

Before: $2x - 5 = 9$

Action: Apply the same inverse addition to both sides.

Operation: $2x - 5 = 9 \rightarrow 2x - 5 + (5) = 9 + (5)$

After: $2x - 5 + (5) = 9 + (5)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl} 2x - 5 & = & 9 \\ + 5 & & + 5 \\ \hline 2x & = & 14 \end{array}$$

Opposite terms sum to zero.

$$2x = 14$$

Apply + 5 to both sides

$2x - 5$ equals 9. Apply + 5 to each side. Result: $2x$ equals 14. The marked opposite terms sum to zero.

Step 3

Before: $-5+(5)$

Action: Cancel the additive inverse pair.

Operation: $-5+(5) \rightarrow -5+(5)=0; 2x+0=9+(5)$

After: $-5+(5)=0; 2x+0=9+(5)$

Why: Opposite added terms become zero. The variable term remains.

$$-5+(5)$$

$$-5+(5)=0; 2x+0=9+(5)$$

Cancel the additive inverse pair.

$-5+(5); -5+(5)=0; 2x+0=9+(5)$

Step 4

Before: $9+(5)$

Action: Calculate the right-hand side.

Operation: $9+(5) \rightarrow 14$

After: 14

Why: Finish this arithmetic before dividing.

$$9+(5)$$

$$14$$

Calculate the right-hand side.

$9+(5); 14$

Step 5

Before: $2x=14$

Action: Divide both sides by the nonzero coefficient.

Operation: $2x=14 \rightarrow (2x)/(2)=(14)/(2)$

After: $(2x)/(2)=(14)/(2)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{2x}{2} = \frac{14}{2}$$

$$\cancel{(2/2)} \times x = 14/(2)$$

$$2/2=1 \text{ because } 2 \neq 0$$

$$x=7$$

Divide both sides; equal nonzero factors make one

Divide $2x=14$ by 2 on each side. The coefficient quotient $2/2$ equals one, leaving $x=7$.

Step 6

Before: $(2/2)x=(14)/(2)$

Action: Reduce the coefficient quotient to one.

Operation: $(2/2)x=(14)/(2) \rightarrow 1 \times x=(14)/(2)$

After: $1 \times x=(14)/(2)$

Why: $2/2=1$, not zero; 2 is nonzero.

$$(2/2)x=(14)/(2)$$

$$1 \times x=(14)/(2)$$

Reduce the coefficient quotient to one.

$(2/2)x=(14)/(2)$; $1 \times x=(14)/(2)$

Step 7

Before: $x=(14)/(2)$

Action: Evaluate the quotient.

Operation: $x=(14)/(2) \rightarrow x=7$

After: $x=7$

Why: Keep an exact fractional value if the division is not integral.

$$x=(14)/(2)$$

$$x=7$$

Evaluate the quotient.

$x=(14)/(2)$; $x=7$

Step 8

Before: $x=7$

Action: Substitute the candidate into the original rule.

Operation: $x=7 \rightarrow h(7)=(2)(7)+(-5)$

After: $h(7)=(2)(7)+(-5)$

Why: Check the rule before applying any contextual restriction.

$$x=7$$

$$h(7)=(2)(7)+(-5)$$

Substitute the candidate into the original rule.

$x=7$; $h(7)=(2)(7)+(-5)$

Step 9

Before: $(2)(7)$

Action: Calculate the variable term.

Operation: $(2)(7) \rightarrow 14$

After: 14

Why: The product recovers the right-hand side before the constant was removed.

$$(2)(7)$$

$$14$$

Calculate the variable term.

$(2)(7)$; 14

Step 10

Before: $14+(-5)$

Action: Restore the constant and compare with the target.

Operation: $14+(-5) \rightarrow 9=9$

After: $9=9$

Why: The algebraic candidate produces the required output.

$$14+(-5)$$

$$9=9$$

Restore the constant and compare with the target.

$14+(-5); 9=9$

Step 11

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow x=7$

After: $x=7$

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$$x=7$$

Report the input with its meaning and unit.

State the requested result; $x=7$

1. The output is known; the input is unknown. $h(x)=9 \rightarrow 2x-5=9$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $2x - 5=9 \rightarrow 2x - 5+(5)=9+(5)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $-5+(5) \rightarrow -5+(5)=0$; $2x+0=9+(5)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $9+(5) \rightarrow 14$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $2x=14 \rightarrow (2x)/(2)=(14)/(2)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(2/2)x=(14)/(2) \rightarrow 1 \times x=(14)/(2)$ $2/2=1$, not zero; 2 is nonzero.
7. Evaluate the quotient. $x=(14)/(2) \rightarrow x=7$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $x=7 \rightarrow h(7)=(2)(7)+(-5)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(2)(7) \rightarrow 14$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $14+(-5) \rightarrow 9=9$ The algebraic candidate produces the required output.
11. Report the input with its meaning and unit. State the requested result $\rightarrow x=7$ The final statement answers the original question after both the algebraic and domain checks.

Answer: $x=7$

$$2x - 5 = 9.$$

Add 5 on both sides:

$$2x - 5 + 5 = 9 + 5.$$

$$2x + 0 = 14; 2x = 14.$$

Divide both sides by 2:

$$(2 \times x)/(2) = 14/(2).$$

$$(2/2) \times x = 7; 1 \times x = 7.$$

$$x = 7.$$

$$\text{Check: } 2 \times (7) - 5 = 14 - 5 = 9.$$

What to look for

- Selects the correct method.
- Shows intermediate calculations.
- Checks notation and allowed inputs.

Hints to offer

Nine is the output; x is unknown.

Set $2x - 5 = 9$ and add five on both sides.

$2x = 14$. Divide both sides by two and check.

With help

Apply the method: teaching example

Find x if $f(x)=3x-2$ and $f(x)=10$.

Step 1

Before: $3x-2=10$

Action: Choose the operation that removes negative two.

After: $3x=$ __ after adding the same amount to both sides.

Why: Then divide both sides by the coefficient.

$$3x - 2 = 10.$$

Add 2 on both sides:

$$3x - 2 + 2 = 10 + 2.$$

$$3x + 0 = 12; 3x = 12.$$

Divide both sides by 3:

$$(3 \times x)/(3) = 12/(3).$$

$$(3/3) \times x = 4; 1 \times x = 4.$$

$$x = 4.$$

$$\text{Check: } 3 \times (4) - 2 = 12 - 2 = 10.$$

Question 4

For $p(x) = -3x + 2$, find $p(-2)$, using a hint only if needed.

Answer and working

Step 1

Before: $p(-2)$

Action: Read the supplied input.

Operation: $p(-2) \rightarrow x = -2$

After: $x = -2$

Why: The number inside the brackets is the input; the required result is the output.

$$p(-2)$$

$$x = -2$$

Read the supplied input.

$p(-2)$; $x = -2$

Step 2

Before: $p(x) = -3x + 2$

Action: Replace each occurrence of the input variable.

Operation: $p(x) = -3x + 2 \rightarrow p(-2) = -3(-2) + 2$

After: $p(-2) = -3(-2) + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$p(x) = -3x + 2$$

$$p(-2) = -3(-2) + 2$$

Replace each occurrence of the input variable.

$$p(x) = -3x + 2; p(-2) = -3(-2) + 2$$

Step 3

Before: $(-3) \times (-2)$

Action: Multiply the coefficient and the input.

Operation: $(-3) \times (-2) \rightarrow 6$

After: 6

Why: Two negative factors give a positive product.

$$(-3) \times (-2)$$

$$6$$

Multiply the coefficient and the input.

$$(-3) \times (-2); 6$$

Step 4

Before: $(6)+(2)$

Action: Apply the constant term.

Operation: $(6)+(2) \rightarrow 8$

After: 8

Why: The constant is added once after the variable terms have been evaluated.

$$(6)+(2)$$

$$8$$

Apply the constant term.

$(6)+(2)$; 8

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow p(-2)=8$

After: $p(-2)=8$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$p(-2)=8$$

Use the original function name.

Label the computed output; $p(-2)=8$

1. Read the supplied input. $p(-2) \rightarrow x=-2$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $p(x)=-3x+2 \rightarrow p(-2)=-3(-2) + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(-3) \times (-2) \rightarrow 6$ Two negative factors give a positive product.
4. Apply the constant term. $(6) + (2) \rightarrow 8$ The constant is added once after the variable terms have been evaluated.
5. Use the original function name. Label the computed output $\rightarrow p(-2)=8$ The requested input has now passed through every operation in the rule.

Answer: $p(-2)=8$

$$p(-2)=-3(-2)+2.$$

Two negative factors give 6.

Then $6+2=8$.

Check the ordered pair $(-2, 8)$ fits the rule.

What to look for

- Selects the correct method.
- Shows intermediate calculations.
- Checks notation and allowed inputs.

Hints to offer

Use the entire input negative two.

Substitute: $-3(-2) + 2$.

The two negative factors give six. Finish $6 + 2$.

Question 5

A table lists (0, 2), (1, 2), (1, 3). Is it a function? Explain the deciding input.

Answer and working

Step 1

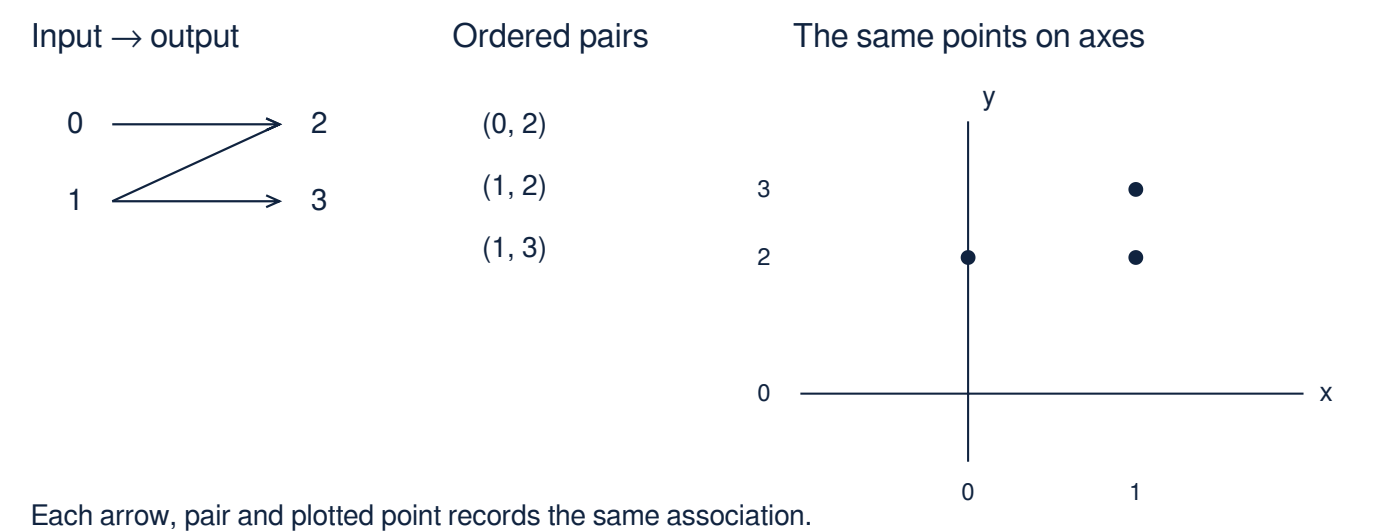
Before: Input set [0, 1]; pairs [(0, 2), (1, 2), (1, 3)]

Action: Interpret each ordered pair as an association.

Operation: Input set [0, 1]; pairs [(0, 2), (1, 2), (1, 3)] → The first coordinate is the input x; the second is its associated output y.

After: The first coordinate is the input x; the second is its associated output y.

Why: A relation is an association between elements of the two sets.



One relation in three representations

Given input set [0, 1], the relation contains pairs [(0, 2), (1, 2), (1, 3)]. The left mapping arrows, ordered-pair list and discrete point graph represent those same pairs; no connecting segments or extra inputs are implied.

Step 2

Before: Input $x=0$

Action: Collect its distinct associated outputs.

Operation: Input $x=0 \rightarrow$ Outputs $[2]$; count 1

After: Outputs $[2]$; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=0$

Outputs $[2]$; count 1

Collect its distinct associated outputs.

Input $x=0$; Outputs $[2]$; count 1

Step 3

Before: Input $x=1$

Action: Collect its distinct associated outputs.

Operation: Input $x=1 \rightarrow$ Outputs $[2, 3]$; count 2

After: Outputs $[2, 3]$; count 2

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=1$

Outputs $[2, 3]$; count 2

Collect its distinct associated outputs.

Input $x=1$; Outputs $[2, 3]$; count 2

Step 4

Before: Apply exactly one output to every stated input

Action: State the decision and its reason.

Operation: Apply exactly one output to every stated input → Not a function on the stated input set: input 1 has 2 distinct outputs

After: Not a function on the stated input set: input 1 has 2 distinct outputs

Why: A single missing or multiple-output input breaks the function condition on the specified set.

Apply exactly one output to every stated input

Not a function on the stated input set: input 1 has 2 distinct outputs

State the decision and its reason.

Apply exactly one output to every stated input; Not a function on the stated input set: input 1 has 2 distinct; outputs

1. Interpret each ordered pair as an association. Input set $[0, 1]$; pairs $[(0, 2), (1, 2), (1, 3)]$ → The first coordinate is the input x ; the second is its associated output y . A relation is an association between elements of the two sets.
2. Collect its distinct associated outputs. Input $x=0$ → Outputs $[2]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
3. Collect its distinct associated outputs. Input $x=1$ → Outputs $[2, 3]$; count 2 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
4. State the decision and its reason. Apply exactly one output to every stated input → Not a function on the stated input set: input 1 has 2 distinct outputs A single missing or multiple-output input breaks the function condition on the specified set.

Answer: Not a function on the stated input set: input 1 has 2 distinct outputs

Input 0 has output 2.

Input 1 has both 2 and 3.

Because the same input has two different outputs, the table is not a function.

What to look for

- Selects the correct method.
- Shows intermediate calculations.
- Checks notation and allowed inputs.

Hints to offer

Check outputs for each input, not inputs for each output.

Group the two rows that begin with one.

Input one has outputs two and three. Compare that with exactly one output.

Question 6

For $C(n)=5n+2$ dollars and whole $n \geq 0$, could the exact cost be \$14?

Answer and working

Step 1

Before: $C(n)=14$

Action: The output is known; the input is unknown.

Operation: $C(n)=14 \rightarrow 5n+2=14$

After: $5n+2=14$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$C(n)=14$$

$$5n+2=14$$

The output is known; the input is unknown.

$C(n)=14$; $5n+2=14$

Step 2

Before: $5n + 2 = 14$

Action: Apply the same inverse addition to both sides.

Operation: $5n + 2 = 14 \rightarrow 5n + 2 + (-2) = 14 + (-2)$

After: $5n + 2 + (-2) = 14 + (-2)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl}
 5n + 2 & = & 14 \\
 - 2 & & - 2 \\
 \hline
 5n & \overset{+2}{\underset{-2}{=}} & 12
 \end{array}$$

Opposite terms sum to zero.

$$5n = 12$$

Apply $- 2$ to both sides

$5n + 2$ equals 14. Apply $- 2$ to each side. Result: $5n$ equals 12. The marked opposite terms sum to zero.

Step 3

Before: $2+(-2)$

Action: Cancel the additive inverse pair.

Operation: $2+(-2) \rightarrow 2+(-2)=0$; $5n+0=14+(-2)$

After: $2+(-2)=0$; $5n+0=14+(-2)$

Why: Opposite added terms become zero. The variable term remains.

$$2+(-2)$$

$$2+(-2)=0; 5n+0=14+(-2)$$

Cancel the additive inverse pair.

$2+(-2)$; $2+(-2)=0$; $5n+0=14+(-2)$

Step 4

Before: $14+(-2)$

Action: Calculate the right-hand side.

Operation: $14+(-2) \rightarrow 12$

After: 12

Why: Finish this arithmetic before dividing.

$$14+(-2)$$

$$12$$

Calculate the right-hand side.

$14+(-2)$; 12

Step 5

Before: $5n=12$

Action: Divide both sides by the nonzero coefficient.

Operation: $5n=12 \rightarrow (5n)/(5)=(12)/(5)$

After: $(5n)/(5)=(12)/(5)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{5n}{5} = \frac{12}{5}$$

$$\cancel{(5/5)} \times n = 12/(5)$$

$$5/5=1 \text{ because } 5 \neq 0$$

$$n=12/5$$

Divide both sides; equal nonzero factors make one

Divide $5n=12$ by 5 on each side. The coefficient quotient $5/5$ equals one, leaving $n=12/5$.

Step 6

Before: $(5/5)n=(12)/(5)$

Action: Reduce the coefficient quotient to one.

Operation: $(5/5)n=(12)/(5) \rightarrow 1 \times n=(12)/(5)$

After: $1 \times n=(12)/(5)$

Why: $5/5=1$, not zero; 5 is nonzero.

$$(5/5)n=(12)/(5)$$

$$1 \times n=(12)/(5)$$

Reduce the coefficient quotient to one.

$(5/5)n=(12)/(5); 1 \times n=(12)/(5)$

Step 7

Before: $n=(12)/(5)$

Action: Evaluate the quotient.

Operation: $n=(12)/(5) \rightarrow n=12/5$

After: $n=12/5$

Why: Keep an exact fractional value if the division is not integral.

$$n=(12)/(5)$$

$$n=12/5$$

Evaluate the quotient.

$n=(12)/(5); n=12/5$

Step 8

Before: $n=12/5$

Action: Substitute the candidate into the original rule.

Operation: $n=12/5 \rightarrow C(12/5)=(5)(12/5)+(2)$

After: $C(12/5)=(5)(12/5)+(2)$

Why: Check the rule before applying any contextual restriction.

$$n=12/5$$

$$C(12/5)=(5)(12/5)+(2)$$

Substitute the candidate into the original rule.

$n=12/5$; $C(12/5)=(5)(12/5)+(2)$

Step 9

Before: $(5)(12/5)$

Action: Calculate the variable term.

Operation: $(5)(12/5) \rightarrow 12$

After: 12

Why: The product recovers the right-hand side before the constant was removed.

$$(5)(12/5)$$

$$12$$

Calculate the variable term.

$(5)(12/5)$; 12

Step 10

Before: $12+(2)$

Action: Restore the constant and compare with the target.

Operation: $12+(2) \rightarrow 14=14$

After: $14=14$

Why: The algebraic candidate produces the required output.

$$12+(2)$$

$$14=14$$

Restore the constant and compare with the target.

$12+(2)$; $14=14$

Step 11

Before: Candidate $n=12/5$

Action: Check the whole-number, nonnegative input condition.

Operation: Candidate $n=12/5 \rightarrow$ Not permitted: this input is not a nonnegative whole number.

After: Not permitted: this input is not a nonnegative whole number.

Why: A fractional number of whole items cannot be ordered.

Candidate $n=12/5$

Not permitted: this input is not a nonnegative whole number.

Check the whole-number, nonnegative input condition.

Candidate $n=12/5$; Not permitted: this input is not a nonnegative whole number.

Step 12

Before: $C(2)=(5)(2)+(2)$

Action: Multiply the whole-item count by the per-item amount.

Operation: $C(2)=(5)(2)+(2) \rightarrow 10+(2)$

After: $10+(2)$

Why: This is a permitted neighbouring count.

$$C(2)=(5)(2)+(2)$$

$$10+(2)$$

Multiply the whole-item count by the per-item amount.

$C(2)=(5)(2)+(2)$; $10+(2)$

Step 13

Before: $10+(2)$

Action: Add the fixed amount.

Operation: $10+(2) \rightarrow C(2)=12$ dollars

After: $C(2)=12$ dollars

Why: Compare this attainable cost with the required exact cost.

$$10+(2)$$

$$C(2)=12 \text{ dollars}$$

Add the fixed amount.

$10+(2)$; $C(2)=12$ dollars

Step 14

Before: $C(3)=(5)(3)+(2)$

Action: Multiply the whole-item count by the per-item amount.

Operation: $C(3)=(5)(3)+(2) \rightarrow 15+(2)$

After: $15+(2)$

Why: This is a permitted neighbouring count.

$$C(3)=(5)(3)+(2)$$

$$15+(2)$$

Multiply the whole-item count by the per-item amount.

$C(3)=(5)(3)+(2)$; $15+(2)$

Step 15

Before: $15+(2)$

Action: Add the fixed amount.

Operation: $15+(2) \rightarrow C(3)=17$ dollars

After: $C(3)=17$ dollars

Why: Compare this attainable cost with the required exact cost.

$$15+(2)$$

$$C(3)=17 \text{ dollars}$$

Add the fixed amount.

$15+(2)$; $C(3)=17$ dollars

Step 16

Before: Both neighbouring whole counts miss the target

Action: State the contextual conclusion.

Operation: Both neighbouring whole counts miss the target → No exact 14-dollar order is available.

After: No exact 14-dollar order is available.

Why: Every additional item changes the cost by the positive fixed per-item amount; no whole count lies between the neighbours.

Both neighbouring whole counts miss the target

No exact 14-dollar order is available.

State the contextual conclusion.

Both neighbouring whole counts miss the target; No exact 14-dollar order is available.

Step 17

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result → No exact 14-dollar order is available.

After: No exact 14-dollar order is available.

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

No exact 14-dollar order is available.

Report the input with its meaning and unit.

State the requested result; No exact 14-dollar order is available.

1. The output is known; the input is unknown. $C(n)=14 \rightarrow 5n+2=14$ Set the rule equal to the known output. Do not substitute the output in place of the input.
 2. Apply the same inverse addition to both sides. $5n + 2=14 \rightarrow 5n + 2+(-2)=14+(-2)$ Equality is preserved because both sides receive the same operation.
 3. Cancel the additive inverse pair. $2+(-2) \rightarrow 2+(-2)=0$; $5n+0=14+(-2)$ Opposite added terms become zero. The variable term remains.
 4. Calculate the right-hand side. $14+(-2) \rightarrow 12$ Finish this arithmetic before dividing.
 5. Divide both sides by the nonzero coefficient. $5n=12 \rightarrow (5n)/(5)=(12)/(5)$ The same nonzero divisor is applied to both sides.
 6. Reduce the coefficient quotient to one. $(5/5)n=(12)/(5) \rightarrow 1 \times n=(12)/(5)$ $5/5=1$, not zero; 5 is nonzero.
 7. Evaluate the quotient. $n=(12)/(5) \rightarrow n=12/5$ Keep an exact fractional value if the division is not integral.
 8. Substitute the candidate into the original rule. $n=12/5 \rightarrow C(12/5)=(5)(12/5)+(2)$ Check the rule before applying any contextual restriction.
 9. Calculate the variable term. $(5)(12/5) \rightarrow 12$ The product recovers the right-hand side before the constant was removed.
 10. Restore the constant and compare with the target. $12+(2) \rightarrow 14=14$ The algebraic candidate produces the required output.
 11. Check the whole-number, nonnegative input condition. Candidate $n=12/5 \rightarrow$ Not permitted: this input is not a nonnegative whole number. A fractional number of whole items cannot be ordered.
 12. Multiply the whole-item count by the per-item amount. $C(2)=(5)(2)+(2) \rightarrow 10+(2)$ This is a permitted neighbouring count.
 13. Add the fixed amount. $10+(2) \rightarrow C(2)=12$ dollars Compare this attainable cost with the required exact cost.
 14. Multiply the whole-item count by the per-item amount. $C(3)=(5)(3)+(2) \rightarrow 15+(2)$ This is a permitted neighbouring count.
 15. Add the fixed amount. $15+(2) \rightarrow C(3)=17$ dollars Compare this attainable cost with the required exact cost.
 16. State the contextual conclusion. Both neighbouring whole counts miss the target $\rightarrow$ No exact 14-dollar order is available. Every additional item changes the cost by the positive fixed per-item amount; no whole count lies between the neighbours.
 17. Report the input with its meaning and unit. State the requested result $\rightarrow$ No exact 14-dollar order is available. The final statement answers the original question after both the algebraic and domain checks.
- Answer: No exact 14-dollar order is available.

$$5n + 2 = 14.$$

Subtract 2 on both sides:

$$5n + 2 - 2 = 14 - 2.$$

$$5n + 0 = 12; 5n = 12.$$

Divide both sides by 5:

$$(5 \times n)/(5) = 12/(5).$$

$$(5/5) \times n = 2.4; 1 \times n = 2.4.$$

$$n = 2.4.$$

$$\text{Check: } 5 \times (2.4) + 2 = 12 + 2 = 14.$$

The equation gives 2.4, but tickets must be whole.

$$C(2) = 5(2) + 2 = 10 + 2 = 12 \text{ dollars.}$$

$$C(3) = 5(3) + 2 = 15 + 2 = 17 \text{ dollars.}$$

No allowed input gives exactly 14 dollars.

What to look for

- Selects the correct method.
- Shows intermediate calculations.
- Checks notation and allowed inputs.

Hints to offer

The cost is given and n must be whole.

Set $5n + 2 = 14$; subtract two on both sides.

Divide $5n = 12$ by five. Decide whether that input is permitted.

On my own

Question 7



Input → rule → output

Input 2; rule $4x - 3$; output ?.

$f(x)=4x - 3$. Find $f(2)$.

Answer and working**Step 1**

Before: $f(2)$

Action: Read the supplied input.

Operation: $f(2) \rightarrow x=2$

After: $x=2$

Why: The number inside the brackets is the input; the required result is the output.

$$f(2)$$

$$x=2$$

Read the supplied input.

$f(2)$; $x=2$

Step 2

Before: $f(x)=4x-3$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=4x-3 \rightarrow f(2)=4(2) + -3$

After: $f(2)=4(2) + -3$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=4x-3$$

$$f(2)=4(2) + -3$$

Replace each occurrence of the input variable.

$$f(x)=4x-3; f(2)=4(2) + -3$$

Step 3

Before: $(4) \times (2)$

Action: Multiply the coefficient and the input.

Operation: $(4) \times (2) \rightarrow 8$

After: 8

Why: Complete multiplication before addition or subtraction.

$$(4) \times (2)$$

$$8$$

Multiply the coefficient and the input.

$$(4) \times (2); 8$$

Step 4

Before: $(8)+(-3)$

Action: Apply the constant term.

Operation: $(8)+(-3) \rightarrow 5$

After: 5

Why: Adding a negative constant is the same as subtracting its positive magnitude.

$$(8)+(-3)$$

$$5$$

Apply the constant term.

$(8)+(-3)$; 5

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(2)=5$

After: $f(2)=5$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(2)=5$$

Use the original function name.

Label the computed output; $f(2)=5$

1. Read the supplied input. $f(2) \rightarrow x=2$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $f(x)=4x-3 \rightarrow f(2)=4(2) + -3$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(4) \times (2) \rightarrow 8$ Complete multiplication before addition or subtraction.
4. Apply the constant term. $(8) + (-3) \rightarrow 5$ Adding a negative constant is the same as subtracting its positive magnitude.
5. Use the original function name. Label the computed output $\rightarrow f(2)=5$ The requested input has now passed through every operation in the rule.

Answer: $f(2)=5$

$$f(2)=4(2)-3=8-3=5.$$

Check two groups of four minus three leaves five.

What to look for

- Uses the relevant method.
- Shows the working.
- Explains a valid check.

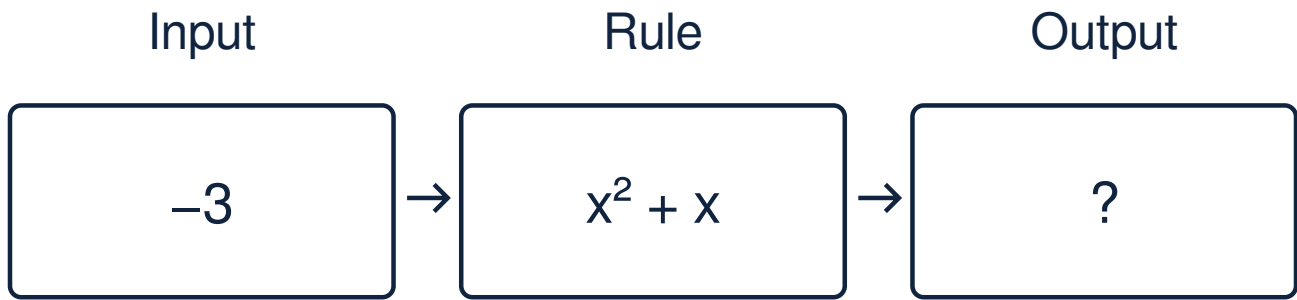
Hints to offer

The input is two.

Replace x with (2) : $4(2) - 3$.

$4 \times 2 = 8$. Subtract three and check.

Question 8



Input $\rightarrow$ rule $\rightarrow$ output

Input -3 ; rule $x^2 + x$; output $?$.

$g(x)=x^2+x$. Find $g(-3)$.

Answer and working**Step 1**

Before: $g(-3)$

Action: Read the supplied input.

Operation: $g(-3) \rightarrow x=-3$

After: $x=-3$

Why: The number inside the brackets is the input; the required result is the output.

$$g(-3)$$

$$x=-3$$

Read the supplied input.

$g(-3)$; $x=-3$

Step 2

Before: $g(x)=x^2+x$

Action: Replace each occurrence of the input variable.

Operation: $g(x)=x^2+x \rightarrow g(-3)=(-3)^2 + (-3)$

After: $g(-3)=(-3)^2 + (-3)$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$g(x)=x^2+x$$

$$g(-3)=(-3)^2 + (-3)$$

Replace each occurrence of the input variable.

$g(x)=x^2+x$; $g(-3)=(-3)^2 + (-3)$

Step 3

Before: $(-3)^2$

Action: Write the square as a product.

Operation: $(-3)^2 \rightarrow (-3) \times (-3)$

After: $(-3) \times (-3)$

Why: Both factors contain the whole input, including its sign.

$$(-3)^2$$

$$(-3) \times (-3)$$

Write the square as a product.

$(-3)^2$; $(-3) \times (-3)$

Step 4

Before: $(-3) \times (-3)$

Action: Multiply the two identical inputs.

Operation: $(-3) \times (-3) \rightarrow 9$

After: 9

Why: Two negative factors give a positive product.

$$(-3) \times (-3)$$

$$9$$

Multiply the two identical inputs.

$(-3) \times (-3); 9$

Step 5

Before: $(1) \times (-3)$

Action: Multiply the coefficient and the input.

Operation: $(1) \times (-3) \rightarrow -3$

After: -3

Why: Complete multiplication before addition or subtraction.

$$(1) \times (-3)$$

$$-3$$

Multiply the coefficient and the input.

$(1) \times (-3); -3$

Step 6

Before: $(9)+(-3)$

Action: Add the two evaluated variable terms.

Operation: $(9)+(-3) \rightarrow 6$

After: 6

Why: The squared input and the unsquared input have different roles; evaluate both before adding.

$$(9)+(-3)$$

$$6$$

Add the two evaluated variable terms.

$(9)+(-3)$; 6

Step 7

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow g(-3)=6$

After: $g(-3)=6$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$g(-3)=6$$

Use the original function name.

Label the computed output; $g(-3)=6$

1. Read the supplied input. $g(-3) \rightarrow x=-3$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $g(x)=x^2+x \rightarrow g(-3)=(-3)^2 + (-3)$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Write the square as a product. $(-3)^2 \rightarrow (-3) \times (-3)$ Both factors contain the whole input, including its sign.
4. Multiply the two identical inputs. $(-3) \times (-3) \rightarrow 9$ Two negative factors give a positive product.
5. Multiply the coefficient and the input. $(1) \times (-3) \rightarrow -3$ Complete multiplication before addition or subtraction.
6. Add the two evaluated variable terms. $(9) + (-3) \rightarrow 6$ The squared input and the unsquared input have different roles; evaluate both before adding.
7. Use the original function name. Label the computed output $\rightarrow g(-3)=6$ The requested input has now passed through every operation in the rule.

Answer: $g(-3)=6$

$$g(-3)=(-3)^2+(-3)=9-3=6.$$

Squaring the whole negative input is essential.

What to look for

- Uses the relevant method.
- Shows the working.
- Explains a valid check.

Hints to offer

Use negative three in both occurrences of x .

Write $(-3)^2 + (-3)$.

The square is nine. Finish $9 + (-3)$.

Question 9

$h(x)=5x-7$. Find the input for output 18.

Answer and working**Step 1**

Before: $h(x)=18$

Action: The output is known; the input is unknown.

Operation: $h(x)=18 \rightarrow 5x-7=18$

After: $5x-7=18$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$h(x)=18$$

$$5x-7=18$$

The output is known; the input is unknown.

$h(x)=18$; $5x-7=18$

Step 2

Before: $5x - 7 = 18$

Action: Apply the same inverse addition to both sides.

Operation: $5x - 7 = 18 \rightarrow 5x - 7 + (7) = 18 + (7)$

After: $5x - 7 + (7) = 18 + (7)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl}
 5x - 7 & = & 18 \\
 + 7 & & + 7 \\
 \hline
 5x & \overset{=7}{\cancel{-7}} \quad \overset{\neq 7}{\cancel{+7}} & = 25
 \end{array}$$

Opposite terms sum to zero.

$$5x = 25$$

Apply + 7 to both sides

$5x - 7$ equals 18. Apply + 7 to each side. Result: $5x$ equals 25. The marked opposite terms sum to zero.

Step 3

Before: $-7+(7)$

Action: Cancel the additive inverse pair.

Operation: $-7+(7) \rightarrow -7+(7)=0; 5x+0=18+(7)$

After: $-7+(7)=0; 5x+0=18+(7)$

Why: Opposite added terms become zero. The variable term remains.

$$-7+(7)$$

$$-7+(7)=0; 5x+0=18+(7)$$

Cancel the additive inverse pair.

$-7+(7); -7+(7)=0; 5x+0=18+(7)$

Step 4

Before: $18+(7)$

Action: Calculate the right-hand side.

Operation: $18+(7) \rightarrow 25$

After: 25

Why: Finish this arithmetic before dividing.

$$18+(7)$$

$$25$$

Calculate the right-hand side.

$18+(7); 25$

Step 5

Before: $5x=25$

Action: Divide both sides by the nonzero coefficient.

Operation: $5x=25 \rightarrow (5x)/(5)=(25)/(5)$

After: $(5x)/(5)=(25)/(5)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{5x}{5} = \frac{25}{5}$$

$$\cancel{(5/5)} \times x = 25/(5)$$

$$5/5=1 \text{ because } 5 \neq 0$$

$$x=5$$

Divide both sides; equal nonzero factors make one

Divide $5x=25$ by 5 on each side. The coefficient quotient $5/5$ equals one, leaving $x=5$.

Step 6

Before: $(5/5)x=(25)/(5)$

Action: Reduce the coefficient quotient to one.

Operation: $(5/5)x=(25)/(5) \rightarrow 1 \times x=(25)/(5)$

After: $1 \times x=(25)/(5)$

Why: $5/5=1$, not zero; 5 is nonzero.

$$(5/5)x=(25)/(5)$$

$$1 \times x=(25)/(5)$$

Reduce the coefficient quotient to one.

$(5/5)x=(25)/(5)$; $1 \times x=(25)/(5)$

Step 7

Before: $x=(25)/(5)$

Action: Evaluate the quotient.

Operation: $x=(25)/(5) \rightarrow x=5$

After: $x=5$

Why: Keep an exact fractional value if the division is not integral.

$$x=(25)/(5)$$

$$x=5$$

Evaluate the quotient.

$x=(25)/(5)$; $x=5$

Step 8

Before: $x=5$

Action: Substitute the candidate into the original rule.

Operation: $x=5 \rightarrow h(5)=(5)(5)+(-7)$

After: $h(5)=(5)(5)+(-7)$

Why: Check the rule before applying any contextual restriction.

$$x=5$$

$$h(5)=(5)(5)+(-7)$$

Substitute the candidate into the original rule.

$x=5$; $h(5)=(5)(5)+(-7)$

Step 9

Before: $(5)(5)$

Action: Calculate the variable term.

Operation: $(5)(5) \rightarrow 25$

After: 25

Why: The product recovers the right-hand side before the constant was removed.

$$(5)(5)$$

$$25$$

Calculate the variable term.

$(5)(5)$; 25

Step 10

Before: $25+(-7)$

Action: Restore the constant and compare with the target.

Operation: $25+(-7) \rightarrow 18=18$

After: $18=18$

Why: The algebraic candidate produces the required output.

$$25+(-7)$$

$$18=18$$

Restore the constant and compare with the target.

$25+(-7); 18=18$

Step 11

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow x=5$

After: $x=5$

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$$x=5$$

Report the input with its meaning and unit.

State the requested result; $x=5$

1. The output is known; the input is unknown. $h(x)=18 \rightarrow 5x-7=18$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $5x - 7=18 \rightarrow 5x - 7+(7)=18+(7)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $-7+(7) \rightarrow -7+(7)=0$; $5x+0=18+(7)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $18+(7) \rightarrow 25$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $5x=25 \rightarrow (5x)/(5)=(25)/(5)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(5/5)x=(25)/(5) \rightarrow 1 \times x=(25)/(5)$ $5/5=1$, not zero; 5 is nonzero.
7. Evaluate the quotient. $x=(25)/(5) \rightarrow x=5$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $x=5 \rightarrow h(5)=(5)(5)+(-7)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(5)(5) \rightarrow 25$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $25+(-7) \rightarrow 18=18$ The algebraic candidate produces the required output.
11. Report the input with its meaning and unit. State the requested result $\rightarrow x=5$ The final statement answers the original question after both the algebraic and domain checks.

Answer: $x=5$

$$5x - 7 = 18.$$

Add 7 on both sides:

$$5x - 7 + 7 = 18 + 7.$$

$$5x + 0 = 25; 5x = 25.$$

Divide both sides by 5:

$$(5 \times x)/(5) = 25/(5).$$

$$(5/5) \times x = 5; 1 \times x = 5.$$

$$x = 5.$$

$$\text{Check: } 5 \times (5) - 7 = 25 - 7 = 18.$$

What to look for

- Uses the relevant method.
- Shows the working.
- Explains a valid check.

Hints to offer

Eighteen is an output.

Set $5x - 7 = 18$; add seven to both sides.

$5x = 25$. Divide both sides by five and substitute to check.

Question 10

$C(n)=6n+4$ dollars, with whole $n \geq 0$. An order costs 28 dollars. How many tickets?

Answer and working**Step 1**

Before: $C(n)=28$

Action: The output is known; the input is unknown.

Operation: $C(n)=28 \rightarrow 6n+4=28$

After: $6n+4=28$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$C(n)=28$$

$$6n+4=28$$

The output is known; the input is unknown.

$C(n)=28$; $6n+4=28$

Step 2

Before: $6n + 4 = 28$

Action: Apply the same inverse addition to both sides.

Operation: $6n + 4 = 28 \rightarrow 6n + 4 + (-4) = 28 + (-4)$

After: $6n + 4 + (-4) = 28 + (-4)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl} 6n + 4 & = & 28 \\ - 4 & & - 4 \\ \hline 6n & = & 24 \end{array}$$

Opposite terms sum to zero.

$$6n = 24$$

Apply -4 to both sides

$6n + 4$ equals 28. Apply -4 to each side. Result: $6n$ equals 24. The marked opposite terms sum to zero.

Step 3

Before: $4+(-4)$

Action: Cancel the additive inverse pair.

Operation: $4+(-4) \rightarrow 4+(-4)=0$; $6n+0=28+(-4)$

After: $4+(-4)=0$; $6n+0=28+(-4)$

Why: Opposite added terms become zero. The variable term remains.

$$4+(-4)$$

$$4+(-4)=0; 6n+0=28+(-4)$$

Cancel the additive inverse pair.

$4+(-4)$; $4+(-4)=0$; $6n+0=28+(-4)$

Step 4

Before: $28+(-4)$

Action: Calculate the right-hand side.

Operation: $28+(-4) \rightarrow 24$

After: 24

Why: Finish this arithmetic before dividing.

$$28+(-4)$$

$$24$$

Calculate the right-hand side.

$28+(-4)$; 24

Step 5

Before: $6n=24$

Action: Divide both sides by the nonzero coefficient.

Operation: $6n=24 \rightarrow (6n)/(6)=(24)/(6)$

After: $(6n)/(6)=(24)/(6)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{6n}{6} = \frac{24}{6}$$

$$\cancel{(6/6)} \times n = 24/(6)$$

$$6/6=1 \text{ because } 6 \neq 0$$

$$n=4$$

Divide both sides; equal nonzero factors make one

Divide $6n=24$ by 6 on each side. The coefficient quotient $6/6$ equals one, leaving $n=4$.

Step 6

Before: $(6/6)n=(24)/(6)$

Action: Reduce the coefficient quotient to one.

Operation: $(6/6)n=(24)/(6) \rightarrow 1 \times n=(24)/(6)$

After: $1 \times n=(24)/(6)$

Why: $6/6=1$, not zero; 6 is nonzero.

$$(6/6)n=(24)/(6)$$

$$1 \times n=(24)/(6)$$

Reduce the coefficient quotient to one.

$(6/6)n=(24)/(6)$; $1 \times n=(24)/(6)$

Step 7

Before: $n=(24)/(6)$

Action: Evaluate the quotient.

Operation: $n=(24)/(6) \rightarrow n=4$

After: $n=4$

Why: Keep an exact fractional value if the division is not integral.

$$n=(24)/(6)$$

$$n=4$$

Evaluate the quotient.

$n=(24)/(6)$; $n=4$

Step 8

Before: $n=4$

Action: Substitute the candidate into the original rule.

Operation: $n=4 \rightarrow C(4)=(6)(4)+(4)$

After: $C(4)=(6)(4)+(4)$

Why: Check the rule before applying any contextual restriction.

$$n=4$$

$$C(4)=(6)(4)+(4)$$

Substitute the candidate into the original rule.

$n=4$; $C(4)=(6)(4)+(4)$

Step 9

Before: $(6)(4)$

Action: Calculate the variable term.

Operation: $(6)(4) \rightarrow 24$

After: 24

Why: The product recovers the right-hand side before the constant was removed.

$$(6)(4)$$

$$24$$

Calculate the variable term.

$(6)(4)$; 24

Step 10

Before: $24+(4)$

Action: Restore the constant and compare with the target.

Operation: $24+(4) \rightarrow 28=28$

After: $28=28$

Why: The algebraic candidate produces the required output.

$$24+(4)$$

$$28=28$$

Restore the constant and compare with the target.

$24+(4)$; $28=28$

Step 11

Before: Candidate $n=4$

Action: Check the whole-number, nonnegative input condition.

Operation: Candidate $n=4 \rightarrow$ Permitted: a nonnegative whole number.

After: Permitted: a nonnegative whole number.

Why: A fractional number of whole items cannot be ordered.

Candidate $n=4$

Permitted: a nonnegative whole number.

Check the whole-number, nonnegative input condition.

Candidate $n=4$; Permitted: a nonnegative whole number.

Step 12

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow n=4$ tickets

After: $n=4$ tickets

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$n=4$ tickets

Report the input with its meaning and unit.

State the requested result; $n=4$ tickets

1. The output is known; the input is unknown. $C(n)=28 \rightarrow 6n+4=28$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $6n + 4=28 \rightarrow 6n + 4+(-4)=28+(-4)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $4+(-4) \rightarrow 4+(-4)=0$; $6n+0=28+(-4)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $28+(-4) \rightarrow 24$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $6n=24 \rightarrow (6n)/(6)=(24)/(6)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(6/6)n=(24)/(6) \rightarrow 1 \times n=(24)/(6)$ $6/6=1$, not zero; 6 is nonzero.
7. Evaluate the quotient. $n=(24)/(6) \rightarrow n=4$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $n=4 \rightarrow C(4)=(6)(4)+(4)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(6)(4) \rightarrow 24$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $24+(4) \rightarrow 28=28$ The algebraic candidate produces the required output.
11. Check the whole-number, nonnegative input condition. Candidate $n=4 \rightarrow$ Permitted: a nonnegative whole number. A fractional number of whole items cannot be ordered.
12. Report the input with its meaning and unit. State the requested result $\rightarrow n=4$ tickets The final statement answers the original question after both the algebraic and domain checks.

Answer: $n=4$ tickets

$$6n + 4 = 28.$$

Subtract 4 on both sides:

$$6n + 4 - 4 = 28 - 4.$$

$$6n + 0 = 24; 6n = 24.$$

Divide both sides by 6:

$$(6 \times n)/(6) = 24/(6).$$

$$(6/6) \times n = 4; 1 \times n = 4.$$

$$n = 4.$$

$$\text{Check: } 6 \times (4) + 4 = 24 + 4 = 28.$$

Four is a nonnegative whole number, so four tickets are allowed and cost exactly 28 dollars.

What to look for

- Uses the relevant method.
- Shows the working.
- Explains a valid check.

Hints to offer

Find the whole-number ticket input.

Set $6n + 4 = 28$; subtract four on both sides.

$6n = 24$. Divide both sides by six, then state the ticket count.

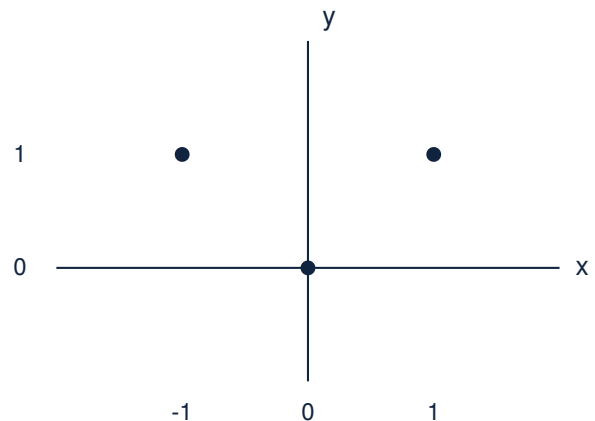
Does $(-1, 1), (0, 0), (1, 1)$ describe a function? Explain.

- A. Yes: every input has one output.
- B. No: two inputs share an output.
- C. No: negative inputs are never allowed.

Step 1

Why: A relation is an association between elements of the two sets.

The same points on axes



Each arrow, pair and plotted point records the same association.

Given input set $[-1, 0, 1]$, the relation contains pairs $\{(-1, 1), (0, 0), (1, 1)\}$. The left mapping arrows, ordered-pair list and discrete point graph represent those same pairs; no connecting segments or extra inputs are implied.

Step 2

Before: Input $x=-1$

Action: Collect its distinct associated outputs.

Operation: Input $x=-1 \rightarrow$ Outputs $[1]$; count 1

After: Outputs $[1]$; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=-1$

Outputs $[1]$; count 1

Collect its distinct associated outputs.

Input $x=-1$; Outputs $[1]$; count 1

Step 3

Before: Input $x=0$

Action: Collect its distinct associated outputs.

Operation: Input $x=0 \rightarrow$ Outputs $[0]$; count 1

After: Outputs $[0]$; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=0$

Outputs $[0]$; count 1

Collect its distinct associated outputs.

Input $x=0$; Outputs $[0]$; count 1

Step 4

Before: Input $x=1$

Action: Collect its distinct associated outputs.

Operation: Input $x=1 \rightarrow$ Outputs $[1]$; count 1

After: Outputs $[1]$; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=1$

Outputs $[1]$; count 1

Collect its distinct associated outputs.

Input $x=1$; Outputs $[1]$; count 1

Step 5

Before: Apply exactly one output to every stated input

Action: State the decision and its reason.

Operation: Apply exactly one output to every stated input $\rightarrow$ Function on the stated input set; $f(-1)=1$, $f(0)=0$, $f(1)=1$

After: Function on the stated input set; $f(-1)=1$, $f(0)=0$, $f(1)=1$

Why: A single missing or multiple-output input breaks the function condition on the specified set.

Apply exactly one output to every stated input

Function on the stated input set; $f(-1)=1$, $f(0)=0$, $f(1)=1$

State the decision and its reason.

Apply exactly one output to every stated input; Function on the stated input set; $f(-1)=1$, $f(0)=0$, $f(1)=1$

1. Interpret each ordered pair as an association. Input set $[-1, 0, 1]$; pairs $[(-1, 1), (0, 0), (1, 1)] \rightarrow$ The first coordinate is the input x ; the second is its associated output y . A relation is an association between elements of the two sets.
2. Collect its distinct associated outputs. Input $x=-1 \rightarrow$ Outputs $[1]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
3. Collect its distinct associated outputs. Input $x=0 \rightarrow$ Outputs $[0]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
4. Collect its distinct associated outputs. Input $x=1 \rightarrow$ Outputs $[1]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
5. State the decision and its reason. Apply exactly one output to every stated input $\rightarrow$ Function on the stated input set; $f(-1)=1, f(0)=0, f(1)=1$ A single missing or multiple-output input breaks the function condition on the specified set.

Answer: Function on the stated input set; $f(-1)=1, f(0)=0, f(1)=1$

Every listed input has one output. -1 and 1 share output 1 , which is allowed.

It is a function on the three listed inputs.

What to look for

- Uses the relevant method.
- Shows the working.
- Explains a valid check.

Hints to offer

A shared output is allowed.

Group rows by their first number.

Each of $-1, 0$ and 1 has just one output. Apply the definition.

Question 12

A learner claims $(x+8)/4 = x+2$. Test $x=4$ and repair the expression.

Answer and working

Step 1

Before: $(x+8)/4 = x+2$?

Action: Test the same input $x=4$ on both sides.

Operation: $(x+8)/4 = x+2$? → Left: $(4+8)/4$; right: $4+2$

After: Left: $(4+8)/4$; right: $4+2$

Why: One failed input disproves a claim stated for every input.

$$(x+8)/4 = x+2?$$

$$\text{Left: } (4+8)/4; \text{ right: } 4+2$$

Test the same input $x=4$ on both sides.

$(x+8)/4 = x+2$?; Left: $(4+8)/4$; right: $4+2$

Step 2

Before: $(4+8)/4$

Action: Add inside the numerator first.

Operation: $(4+8)/4 \rightarrow 12/4$

After: $12/4$

Why: The fraction bar groups the entire sum.

$$(4+8)/4$$

$$12/4$$

Add inside the numerator first.

$(4+8)/4$; $12/4$

Step 3

Before: $12/4$

Action: Divide the whole numerator by four.

Operation: $12/4 \rightarrow 3$

After: 3

Why: Division applies after the numerator is computed.

$$12/4$$

$$3$$

Divide the whole numerator by four.

$12/4$; 3

Step 4

Before: $4+2$

Action: Evaluate the claimed right side.

Operation: $4+2 \rightarrow 6$

After: 6

Why: This is a separate calculation with the same input.

$$4+2$$

$$6$$

Evaluate the claimed right side.

$4+2$; 6

Step 5

Before: Left 3, right 6

Action: Compare the two results.

Operation: Left 3, right 6 $\rightarrow 3 \neq 6$; the claimed identity fails

After: $3 \neq 6$; the claimed identity fails

Why: The first term was not divided by four in the claimed expression.

Left 3, right 6

$3 \neq 6$; the claimed identity fails

Compare the two results.

Left 3, right 6; $3 \neq 6$; the claimed identity fails

Step 6

Before: $(x+8)/4$

Action: Distribute division across both added terms.

Operation: $(x+8)/4 \rightarrow x/4+8/4$

After: $x/4+8/4$

Why: For a nonzero common denominator, every numerator term is divided by it.

$$(x+8)/4$$

$$x/4+8/4$$

Distribute division across both added terms.

$(x+8)/4$; $x/4+8/4$

Step 7

Before: $8/4$

Action: Reduce only the numerical quotient.

Operation: $8/4 \rightarrow 2$; therefore $(x+8)/4=x/4+2$

After: 2 ; therefore $(x+8)/4=x/4+2$

Why: $x/4$ remains $x/4$; it does not become x .

$$8/4$$

$$2; \text{ therefore } (x+8)/4=x/4+2$$

Reduce only the numerical quotient.

$8/4$; 2 ; therefore $(x+8)/4=x/4+2$

Step 8

Before: $x/4+2$ with $x=4$

Action: Check the repaired expression.

Operation: $x/4+2$ with $x=4 \rightarrow 4/4+2$

After: $4/4+2$

Why: Use the same deciding input as before.

$$x/4+2 \text{ with } x=4$$

$$4/4+2$$

Check the repaired expression.

$x/4+2$ with $x=4$; $4/4+2$

Step 9

Before: $4/4+2$

Action: Complete division before addition.

Operation: $4/4+2 \rightarrow 1+2=3$

After: $1+2=3$

Why: The repaired form now agrees with the original left side.

$$4/4+2$$

$$1+2=3$$

Complete division before addition.

$4/4+2$; $1+2=3$

1. Test the same input $x=4$ on both sides. $(x+8)/4 = x+2?$ → Left: $(4+8)/4$; right: $4+2$ One failed input disproves a claim stated for every input.
 2. Add inside the numerator first. $(4+8)/4 \rightarrow 12/4$ The fraction bar groups the entire sum.
 3. Divide the whole numerator by four. $12/4 \rightarrow 3$ Division applies after the numerator is computed.
 4. Evaluate the claimed right side. $4+2 \rightarrow 6$ This is a separate calculation with the same input.
 5. Compare the two results. Left 3, right 6 → $3 \neq 6$; the claimed identity fails The first term was not divided by four in the claimed expression.
 6. Distribute division across both added terms. $(x+8)/4 \rightarrow x/4+8/4$ For a nonzero common denominator, every numerator term is divided by it.
 7. Reduce only the numerical quotient. $8/4 \rightarrow 2$; therefore $(x+8)/4 = x/4+2$ $x/4$ remains $x/4$; it does not become x .
 8. Check the repaired expression. $x/4+2$ with $x=4 \rightarrow 4/4+2$ Use the same deciding input as before.
 9. Complete division before addition. $4/4+2 \rightarrow 1+2=3$ The repaired form now agrees with the original left side.
- Answer: The claim is false. The correct identity is $(x+8)/4 = x/4+2$.

Left: $(4+8)/4 = 12/4 = 3$.

Claimed right: $4+2=6$.

They differ.

Divide both terms: $(x+8)/4 = x/4+8/4 = x/4+2$.

At $x = 4$ this gives $1+2=3$.

What to look for

- Uses the relevant method.
- Shows the working.
- Explains a valid check.

Hints to offer

Test both expressions at the same input.

At $x = 4$, the left is $12/4$ and the claimed right is $4 + 2$.

They differ. Divide each numerator term by four: $x/4 + 8/4$. Simplify the constant.

Come back

Question 13

$f(x)=2x+5$. Find $f(0)$.

Answer and working**Step 1**

Before: $f(0)$

Action: Read the supplied input.

Operation: $f(0) \rightarrow x=0$

After: $x=0$

Why: The number inside the brackets is the input; the required result is the output.

$$f(0)$$

$$x=0$$

Read the supplied input.

$f(0)$; $x=0$

Step 2

Before: $f(x)=2x+5$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=2x+5 \rightarrow f(0)=2(0) + 5$

After: $f(0)=2(0) + 5$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=2x+5$$

$$f(0)=2(0) + 5$$

Replace each occurrence of the input variable.

$$f(x)=2x+5; f(0)=2(0) + 5$$

Step 3

Before: $(2) \times (0)$

Action: Multiply the coefficient and the input.

Operation: $(2) \times (0) \rightarrow 0$

Why: Complete multiplication before addition or subtraction.

$$(2) \times (0)$$

$$0$$

Multiply the coefficient and the input.

$$(2) \times (0); 0$$

Step 4

Before: $(0)+(5)$

Action: Apply the constant term.

Operation: $(0)+(5) \rightarrow 5$

After: 5

Why: The constant is added once after the variable terms have been evaluated.

$$(0)+(5)$$

$$5$$

Apply the constant term.

$(0)+(5); 5$

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(0)=5$

After: $f(0)=5$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(0)=5$$

Use the original function name.

Label the computed output; $f(0)=5$

1. Read the supplied input. $f(0) \rightarrow x=0$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $f(x)=2x+5 \rightarrow f(0)=2(0) + 5$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(2) \times (0) \rightarrow 0$ Complete multiplication before addition or subtraction.
4. Apply the constant term. $(0)+(5) \rightarrow 5$ The constant is added once after the variable terms have been evaluated.
5. Use the original function name. Label the computed output $\rightarrow f(0)=5$ The requested input has now passed through every operation in the rule.

Answer: $f(0)=5$

$$f(0)=2(0)+5=0+5=5.$$

Zero input removes the variable term, not the constant.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Zero is the input.

Replace x with zero in $2x + 5$.

$2 \times 0 = 0$. Keep the constant and finish the sum.

Question 14

$f(x)=2x+5$. Find $f(-4)$.

Answer and working**Step 1**

Before: $f(-4)$

Action: Read the supplied input.

Operation: $f(-4) \rightarrow x=-4$

After: $x=-4$

Why: The number inside the brackets is the input; the required result is the output.

$$f(-4)$$

$$x=-4$$

Read the supplied input.

$f(-4)$; $x=-4$

Step 2

Before: $f(x)=2x+5$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=2x+5 \rightarrow f(-4)=2(-4) + 5$

After: $f(-4)=2(-4) + 5$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=2x+5$$

$$f(-4)=2(-4) + 5$$

Replace each occurrence of the input variable.

$f(x)=2x+5$; $f(-4)=2(-4) + 5$

Step 3

Before: $(2) \times (-4)$

Action: Multiply the coefficient and the input.

Operation: $(2) \times (-4) \rightarrow -8$

After: -8

Why: Complete multiplication before addition or subtraction.

$$(2) \times (-4)$$

$$-8$$

Multiply the coefficient and the input.

$(2) \times (-4)$; -8

Step 4

Before: $(-8)+(5)$

Action: Apply the constant term.

Operation: $(-8)+(5) \rightarrow -3$

After: -3

Why: The constant is added once after the variable terms have been evaluated.

$$(-8)+(5)$$

$$-3$$

Apply the constant term.

$(-8)+(5); -3$

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(-4)=-3$

After: $f(-4)=-3$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(-4)=-3$$

Use the original function name.

Label the computed output; $f(-4)=-3$

1. Read the supplied input. $f(-4) \rightarrow x=-4$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $f(x)=2x+5 \rightarrow f(-4)=2(-4) + 5$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(2) \times (-4) \rightarrow -8$ Complete multiplication before addition or subtraction.
4. Apply the constant term. $(-8)+(5) \rightarrow -3$ The constant is added once after the variable terms have been evaluated.
5. Use the original function name. Label the computed output $\rightarrow f(-4)=-3$ The requested input has now passed through every operation in the rule.

Answer: $f(-4)=-3$

$$f(-4)=2(-4)+5=-8+5=-3.$$

Check: from -8 move five units right to -3 .

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

The input is negative four.

Write $2(-4) + 5$.

Two groups of negative four give -8 . Add five, then check.

Question 15

$g(x)=x^2+1$. Find $g(-5)$.

Answer and working

Step 1

Before: $g(-5)$

Action: Read the supplied input.

Operation: $g(-5) \rightarrow x=-5$

After: $x=-5$

Why: The number inside the brackets is the input; the required result is the output.

$$g(-5)$$

$$x=-5$$

Read the supplied input.

$g(-5); x=-5$

Step 2

Before: $g(x)=x^2+1$

Action: Replace each occurrence of the input variable.

Operation: $g(x)=x^2+1 \rightarrow g(-5)=(-5)^2 + 1$

After: $g(-5)=(-5)^2 + 1$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$g(x)=x^2+1$$

$$g(-5)=(-5)^2 + 1$$

Replace each occurrence of the input variable.

$g(x)=x^2+1; g(-5)=(-5)^2 + 1$

Step 3

Before: $(-5)^2$

Action: Write the square as a product.

Operation: $(-5)^2 \rightarrow (-5) \times (-5)$

After: $(-5) \times (-5)$

Why: Both factors contain the whole input, including its sign.

$$(-5)^2$$

$$(-5) \times (-5)$$

Write the square as a product.

$(-5)^2$; $(-5) \times (-5)$

Step 4

Before: $(-5) \times (-5)$

Action: Multiply the two identical inputs.

Operation: $(-5) \times (-5) \rightarrow 25$

After: 25

Why: Two negative factors give a positive product.

$$(-5) \times (-5)$$

$$25$$

Multiply the two identical inputs.

$(-5) \times (-5)$; 25

Step 5

Before: $(25)+(1)$

Action: Apply the constant term.

Operation: $(25)+(1) \rightarrow 26$

After: 26

Why: The constant is added once after the variable terms have been evaluated.

$$(25)+(1)$$

$$26$$

Apply the constant term.

$(25)+(1)$; 26

Step 6

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow g(-5)=26$

After: $g(-5)=26$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$g(-5)=26$$

Use the original function name.

Label the computed output; $g(-5)=26$

1. Read the supplied input. $g(-5) \rightarrow x=-5$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $g(x)=x^2+1 \rightarrow g(-5)=(-5)^2 + 1$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Write the square as a product. $(-5)^2 \rightarrow (-5) \times (-5)$ Both factors contain the whole input, including its sign.
4. Multiply the two identical inputs. $(-5) \times (-5) \rightarrow 25$ Two negative factors give a positive product.
5. Apply the constant term. $(25)+(1) \rightarrow 26$ The constant is added once after the variable terms have been evaluated.
6. Use the original function name. Label the computed output $\rightarrow g(-5)=26$ The requested input has now passed through every operation in the rule.

Answer: $g(-5)=26$

$$g(-5)=(-5)^2+1=(-5)(-5)+1=25+1=26.$$

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

The entire negative input is squared.

Write $(-5)^2 + 1$.

$(-5)(-5) = 25$. Add the constant.

Question 16

$g(x)=x^2-3x$. Find $g(2)$.

Answer and working**Step 1**

Before: $g(2)$

Action: Read the input.

Operation: $g(2) \rightarrow x=2$

After: $x=2$

Why: Both appearances of x receive the same whole input.

$$g(2)$$

$$x=2$$

Read the input.

$g(2); x=2$

Step 2

Before: $g(x)=x^2-3x$

Action: Substitute both input occurrences.

Operation: $g(x)=x^2-3x \rightarrow g(2)=(2)^2-3(2)$

After: $g(2)=(2)^2-3(2)$

Why: Keep the subtraction in the original rule visible.

$$g(x)=x^2-3x$$

$$g(2)=(2)^2-3(2)$$

Substitute both input occurrences.

$$g(x)=x^2-3x; g(2)=(2)^2-3(2)$$

Step 3

Before: $(2)^2$

Action: Write the square as repeated multiplication.

Operation: $(2)^2 \rightarrow (2)(2)$

After: $(2)(2)$

Why: The sign is inside both factors.

$$(2)^2$$

$$(2)(2)$$

Write the square as repeated multiplication.

$$(2)^2; (2)(2)$$

Step 4

Before: $(2)(2)$

Action: Evaluate the square.

Operation: $(2)(2) \rightarrow 4$

After: 4

Why: Two positive factors produce a positive product.

$$(2)(2)$$

$$4$$

Evaluate the square.

$(2)(2)$; 4

Step 5

Before: $3(2)$

Action: Evaluate the separate linear product.

Operation: $3(2) \rightarrow 6$

After: 6

Why: The original minus sign still stands before this whole product.

$$3(2)$$

$$6$$

Evaluate the separate linear product.

$3(2)$; 6

Step 6

Before: $4-(6)$

Action: Apply the original subtraction.

Operation: $4-(6) \rightarrow 4-6$

After: $4-6$

Why: The subtraction produces a negative result because six is greater than four.

$$4-(6)$$

$$4-6$$

Apply the original subtraction.

$4-(6)$; $4-6$

Step 7

Before: $4-6$

Action: Calculate the final output.

Operation: $4-6 \rightarrow -2$

After: -2

Why: Only now are the two evaluated parts combined.

$$4-6$$

$$-2$$

Calculate the final output.

$4-6$; -2

Step 8

Before: Label the result

Action: Write the original function statement.

Operation: Label the result $\rightarrow g(2)=-2$

After: $g(2)=-2$

Why: This output corresponds to the stated input.

Label the result

$$g(2)=-2$$

Write the original function statement.

Label the result; $g(2)=-2$

1. Read the input. $g(2) \rightarrow x=2$ Both appearances of x receive the same whole input.
2. Substitute both input occurrences. $g(x)=x^2-3x \rightarrow g(2)=(2)^2-3(2)$ Keep the subtraction in the original rule visible.
3. Write the square as repeated multiplication. $(2)^2 \rightarrow (2)(2)$ The sign is inside both factors.
4. Evaluate the square. $(2)(2) \rightarrow 4$ Two positive factors produce a positive product.
5. Evaluate the separate linear product. $3(2) \rightarrow 6$ The original minus sign still stands before this whole product.
6. Apply the original subtraction. $4-(6) \rightarrow 4-6$ The subtraction produces a negative result because six is greater than four.
7. Calculate the final output. $4-6 \rightarrow -2$ Only now are the two evaluated parts combined.
8. Write the original function statement. Label the result $\rightarrow g(2)=-2$ This output corresponds to the stated input.

Answer: $g(2)=-2$

$$g(2)=2^2-3(2)=4-6=-2.$$

Evaluate the square and product before subtraction.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Use two in both places.

Write $2^2 - 3(2)$.

The square is four and the product is six. Finish $4 - 6$.

Question 17

$g(x)=x^2-3x$. Find $g(-2)$.

Answer and working**Step 1**

Before: $g(-2)$

Action: Read the input.

Operation: $g(-2) \rightarrow x=-2$

After: $x=-2$

Why: Both appearances of x receive the same whole input.

$$g(-2)$$

$$x=-2$$

Read the input.

$g(-2); x=-2$

Step 2

Before: $g(x)=x^2-3x$

Action: Substitute both input occurrences.

Operation: $g(x)=x^2-3x \rightarrow g(-2)=(-2)^2-3(-2)$

After: $g(-2)=(-2)^2-3(-2)$

Why: Keep the subtraction in the original rule visible.

$$g(x)=x^2-3x$$

$$g(-2)=(-2)^2-3(-2)$$

Substitute both input occurrences.

$$g(x)=x^2-3x; g(-2)=(-2)^2-3(-2)$$

Step 3

Before: $(-2)^2$

Action: Write the square as repeated multiplication.

Operation: $(-2)^2 \rightarrow (-2)(-2)$

After: $(-2)(-2)$

Why: The sign is inside both factors.

$$(-2)^2$$

$$(-2)(-2)$$

Write the square as repeated multiplication.

$$(-2)^2; (-2)(-2)$$

Step 4

Before: $(-2)(-2)$

Action: Evaluate the square.

Operation: $(-2)(-2) \rightarrow 4$

After: 4

Why: Two negative factors produce a positive product.

$$(-2)(-2)$$

$$4$$

Evaluate the square.

$(-2)(-2)$; 4

Step 5

Before: $3(-2)$

Action: Evaluate the separate linear product.

Operation: $3(-2) \rightarrow -6$

After: -6

Why: The original minus sign still stands before this whole product.

$$3(-2)$$

$$-6$$

Evaluate the separate linear product.

$3(-2)$; -6

Step 6

Before: $4 - (-6)$

Action: Apply the original subtraction.

Operation: $4 - (-6) \rightarrow 4 + 6$

After: $4 + 6$

Why: Subtracting a negative is adding its opposite.

$$4 - (-6)$$

$$4 + 6$$

Apply the original subtraction.

$4 - (-6)$; $4 + 6$

Step 7

Before: $4 + 6$

Action: Calculate the final output.

Operation: $4 + 6 \rightarrow 10$

After: 10

Why: Only now are the two evaluated parts combined.

$$4 + 6$$

$$10$$

Calculate the final output.

$4 + 6$; 10

Step 8

Before: Label the result

Action: Write the original function statement.

Operation: Label the result $\rightarrow g(-2)=10$

After: $g(-2)=10$

Why: This output corresponds to the stated input.

Label the result

$$g(-2)=10$$

Write the original function statement.

Label the result; $g(-2)=10$

1. Read the input. $g(-2) \rightarrow x=-2$ Both appearances of x receive the same whole input.
2. Substitute both input occurrences. $g(x)=x^2-3x \rightarrow g(-2)=(-2)^2-3(-2)$ Keep the subtraction in the original rule visible.
3. Write the square as repeated multiplication. $(-2)^2 \rightarrow (-2)(-2)$ The sign is inside both factors.
4. Evaluate the square. $(-2)(-2) \rightarrow 4$ Two negative factors produce a positive product.
5. Evaluate the separate linear product. $3(-2) \rightarrow -6$ The original minus sign still stands before this whole product.
6. Apply the original subtraction. $4-(-6) \rightarrow 4+6$ Subtracting a negative is adding its opposite.
7. Calculate the final output. $4+6 \rightarrow 10$ Only now are the two evaluated parts combined.
8. Write the original function statement. Label the result $\rightarrow g(-2)=10$ This output corresponds to the stated input.

Answer: $g(-2)=10$

$$g(-2)=(-2)^2-3(-2)=4-(-6)=4+6=10.$$

Subtracting a negative adds its opposite.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Keep negative two together in brackets.

Write $(-2)^2 - 3(-2)$.

The square is four and the product is -6 . Subtracting -6 adds six.

Question 18

$p(x)=6x - 1$. Find $p(1/2)$.

Answer and working**Step 1**

Before: $p(1/2)$

Action: Read the supplied input.

Operation: $p(1/2) \rightarrow x=1/2$

After: $x=1/2$

Why: The number inside the brackets is the input; the required result is the output.

$$p(1/2)$$

$$x=1/2$$

Read the supplied input.

$p(1/2); x=1/2$

Step 2

Before: $p(x)=6x-1$

Action: Replace each occurrence of the input variable.

Operation: $p(x)=6x-1 \rightarrow p(1/2)=6(1/2) + -1$

After: $p(1/2)=6(1/2) + -1$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$p(x)=6x-1$$

$$p(1/2)=6(1/2) + -1$$

Replace each occurrence of the input variable.

$p(x)=6x-1$; $p(1/2)=6(1/2) + -1$

Step 3

Before: $(6) \times (1/2)$

Action: Write the integer as a fraction and multiply numerators.

Operation: $(6) \times (1/2) \rightarrow (6 \times 1)/2 = 6/2$

After: $(6 \times 1)/2 = 6/2$

Why: The denominator counts the equal fractional parts.

$$(6) \times (1/2)$$

$$(6 \times 1)/2 = 6/2$$

Write the integer as a fraction and multiply numerators.

$(6) \times (1/2)$; $(6 \times 1)/2 = 6/2$

Step 4

Before: $6/2$

Action: Divide the numerator by the denominator.

Operation: $6/2 \rightarrow 3$

After: 3

Why: This finishes the variable term before the constant is applied.

$$6/2$$

$$3$$

Divide the numerator by the denominator.

$6/2$; 3

Step 5

Before: $(3)+(-1)$

Action: Apply the constant term.

Operation: $(3)+(-1) \rightarrow 2$

After: 2

Why: Adding a negative constant is the same as subtracting its positive magnitude.

$$(3)+(-1)$$

$$2$$

Apply the constant term.

$(3)+(-1)$; 2

Step 6

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow p(1/2)=2$

After: $p(1/2)=2$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$p(1/2)=2$$

Use the original function name.

Label the computed output; $p(1/2)=2$

1. Read the supplied input. $p(1/2) \rightarrow x=1/2$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $p(x)=6x-1 \rightarrow p(1/2)=6(1/2) + -1$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Write the integer as a fraction and multiply numerators. $(6) \times (1/2) \rightarrow (6 \times 1)/2 = 6/2$ The denominator counts the equal fractional parts.
4. Divide the numerator by the denominator. $6/2 \rightarrow 3$ This finishes the variable term before the constant is applied.
5. Apply the constant term. $(3) + (-1) \rightarrow 2$ Adding a negative constant is the same as subtracting its positive magnitude.
6. Use the original function name. Label the computed output $\rightarrow p(1/2)=2$ The requested input has now passed through every operation in the rule.

Answer: $p(1/2)=2$

$$p(1/2)=6(1/2)-1=(6/2)-1=3-1=2.$$

Six halves make three.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

One half is the input.

Replace x by $1/2$ in $6x - 1$.

Six halves make three wholes. Subtract one.

Question 19

$f(x)=3x+1$. Decide whether $f(2)$ means “find the input” or “find the output”, then solve.

Answer and working

Step 1

Before: $f(2)$

Action: Decide which quantity is already given.

Operation: $f(2) \rightarrow$ Input 2 is known; find its output

After: Input 2 is known; find its output

Why: The bracket supplies the input. In contrast, $f(x)=2$ would specify an output.

$f(2)$

Input 2 is known; find its output

Decide which quantity is already given.

$f(2)$; Input 2 is known; find its output

Step 2

Before: $f(2)$

Action: Read the supplied input.

Operation: $f(2) \rightarrow x=2$

After: $x=2$

Why: The number inside the brackets is the input; the required result is the output.

$$f(2)$$

$$x=2$$

Read the supplied input.

$f(2); x=2$

Step 3

Before: $f(x)=3x+1$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=3x+1 \rightarrow f(2)=3(2) + 1$

After: $f(2)=3(2) + 1$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=3x+1$$

$$f(2)=3(2) + 1$$

Replace each occurrence of the input variable.

$f(x)=3x+1; f(2)=3(2) + 1$

Step 4

Before: $(3) \times (2)$

Action: Multiply the coefficient and the input.

Operation: $(3) \times (2) \rightarrow 6$

After: 6

Why: Complete multiplication before addition or subtraction.

$$(3) \times (2)$$

$$6$$

Multiply the coefficient and the input.

$(3) \times (2); 6$

Step 5

Before: $(6) + (1)$

Action: Apply the constant term.

Operation: $(6) + (1) \rightarrow 7$

After: 7

Why: The constant is added once after the variable terms have been evaluated.

$$(6) + (1)$$

$$7$$

Apply the constant term.

$(6) + (1); 7$

Step 6

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(2)=7$

After: $f(2)=7$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(2)=7$$

Use the original function name.

Label the computed output; $f(2)=7$

1. Decide which quantity is already given. $f(2) \rightarrow$ Input 2 is known; find its output The bracket supplies the input. In contrast, $f(x)=2$ would specify an output.
2. Read the supplied input. $f(2) \rightarrow x=2$ The number inside the brackets is the input; the required result is the output.
3. Replace each occurrence of the input variable. $f(x)=3x+1 \rightarrow f(2)=3(2) + 1$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
4. Multiply the coefficient and the input. $(3) \times (2) \rightarrow 6$ Complete multiplication before addition or subtraction.
5. Apply the constant term. $(6)+(1) \rightarrow 7$ The constant is added once after the variable terms have been evaluated.
6. Use the original function name. Label the computed output $\rightarrow f(2)=7$ The requested input has now passed through every operation in the rule.

Answer: $f(2)=7$

The bracket gives input 2.

Find output: $f(2)=3(2)+1=6+1=7$.

It does not ask for $3x+1=2$.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

The number inside $f(2)$ is the input.

Substitute two into $3x + 1$.

$3 \times 2 = 6$. Add one and name the output.

Question 20

For the same $f(x)=3x+1$, find x if $f(x)=7$.

Answer and working

Step 1

Before: $f(x)=7$

Action: The output is known; the input is unknown.

Operation: $f(x)=7 \rightarrow 3x+1=7$

After: $3x+1=7$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$f(x)=7$$

$$3x+1=7$$

The output is known; the input is unknown.

$f(x)=7$; $3x+1=7$

Step 2

Before: $3x + 1 = 7$

Action: Apply the same inverse addition to both sides.

Operation: $3x + 1 = 7 \rightarrow 3x + 1 + (-1) = 7 + (-1)$

After: $3x + 1 + (-1) = 7 + (-1)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl} 3x + 1 & = & 7 \\ - 1 & & - 1 \\ \hline 3x & = & 6 \end{array}$$

Opposite terms sum to zero.

$$3x = 6$$

Apply -1 to both sides

$3x + 1$ equals 7. Apply -1 to each side. Result: $3x$ equals 6. The marked opposite terms sum to zero.

Step 3

Before: $1+(-1)$

Action: Cancel the additive inverse pair.

Operation: $1+(-1) \rightarrow 1+(-1)=0$; $3x+0=7+(-1)$

After: $1+(-1)=0$; $3x+0=7+(-1)$

Why: Opposite added terms become zero. The variable term remains.

$$1+(-1)$$

$$1+(-1)=0; 3x+0=7+(-1)$$

Cancel the additive inverse pair.

$1+(-1)$; $1+(-1)=0$; $3x+0=7+(-1)$

Step 4

Before: $7+(-1)$

Action: Calculate the right-hand side.

Operation: $7+(-1) \rightarrow 6$

After: 6

Why: Finish this arithmetic before dividing.

$$7+(-1)$$

$$6$$

Calculate the right-hand side.

$7+(-1)$; 6

Step 5

Before: $3x=6$

Action: Divide both sides by the nonzero coefficient.

Operation: $3x=6 \rightarrow (3x)/(3)=(6)/(3)$

After: $(3x)/(3)=(6)/(3)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{3x}{3} = \frac{6}{3}$$

$$\cancel{(3/3)} \times x = 6/(3)$$

$3/3=1$ because $3 \neq 0$

$$x=2$$

Divide both sides; equal nonzero factors make one

Divide $3x=6$ by 3 on each side. The coefficient quotient $3/3$ equals one, leaving $x=2$.

Step 6

Before: $(3/3)x=(6)/(3)$

Action: Reduce the coefficient quotient to one.

Operation: $(3/3)x=(6)/(3) \rightarrow 1 \times x=(6)/(3)$

After: $1 \times x=(6)/(3)$

Why: $3/3=1$, not zero; 3 is nonzero.

$$(3/3)x=(6)/(3)$$

$$1 \times x=(6)/(3)$$

Reduce the coefficient quotient to one.

$(3/3)x=(6)/(3)$; $1 \times x=(6)/(3)$

Step 7

Before: $x=(6)/(3)$

Action: Evaluate the quotient.

Operation: $x=(6)/(3) \rightarrow x=2$

After: $x=2$

Why: Keep an exact fractional value if the division is not integral.

$$x=(6)/(3)$$

$$x=2$$

Evaluate the quotient.

$x=(6)/(3)$; $x=2$

Step 8

Before: $x=2$

Action: Substitute the candidate into the original rule.

Operation: $x=2 \rightarrow f(2)=(3)(2)+(1)$

After: $f(2)=(3)(2)+(1)$

Why: Check the rule before applying any contextual restriction.

$$x=2$$

$$f(2)=(3)(2)+(1)$$

Substitute the candidate into the original rule.

$x=2$; $f(2)=(3)(2)+(1)$

Step 9

Before: $(3)(2)$

Action: Calculate the variable term.

Operation: $(3)(2) \rightarrow 6$

After: 6

Why: The product recovers the right-hand side before the constant was removed.

$$(3)(2)$$

$$6$$

Calculate the variable term.

$(3)(2)$; 6

Step 10

Before: $6+(1)$

Action: Restore the constant and compare with the target.

Operation: $6+(1) \rightarrow 7=7$

After: $7=7$

Why: The algebraic candidate produces the required output.

$$6+(1)$$

$$7=7$$

Restore the constant and compare with the target.

$6+(1); 7=7$

Step 11

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow x=2$

After: $x=2$

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$$x=2$$

Report the input with its meaning and unit.

State the requested result; $x=2$

1. The output is known; the input is unknown. $f(x)=7 \rightarrow 3x+1=7$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $3x + 1=7 \rightarrow 3x + 1+(-1)=7+(-1)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $1+(-1) \rightarrow 1+(-1)=0$; $3x+0=7+(-1)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $7+(-1) \rightarrow 6$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $3x=6 \rightarrow (3x)/(3)=(6)/(3)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(3/3)x=(6)/(3) \rightarrow 1 \times x=(6)/(3)$ $3/3=1$, not zero; 3 is nonzero.
7. Evaluate the quotient. $x=(6)/(3) \rightarrow x=2$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $x=2 \rightarrow f(2)=(3)(2)+(1)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(3)(2) \rightarrow 6$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $6+(1) \rightarrow 7=7$ The algebraic candidate produces the required output.
11. Report the input with its meaning and unit. State the requested result $\rightarrow x=2$ The final statement answers the original question after both the algebraic and domain checks.

Answer: $x=2$

$$3x + 1 = 7.$$

Subtract 1 on both sides:

$$3x + 1 - 1 = 7 - 1.$$

$$3x + 0 = 6; 3x = 6.$$

Divide both sides by 3:

$$(3 \times x)/(3) = 6/(3).$$

$$(3/3) \times x = 2; 1 \times x = 2.$$

$$x = 2.$$

$$\text{Check: } 3 \times (2) + 1 = 6 + 1 = 7.$$

The unknown input is two; the specified output was seven.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Seven is the output, not the input.

Set $3x + 1 = 7$ and subtract one on both sides.

$3x = 6$. Divide both sides by three and check.

Question 21

$h(x) = 4x - 9$. Find x when $h(x) = 11$.

Answer and working

Step 1

Before: $h(x) = 11$

Action: The output is known; the input is unknown.

Operation: $h(x) = 11 \rightarrow 4x - 9 = 11$

After: $4x - 9 = 11$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$h(x) = 11$$

$$4x - 9 = 11$$

The output is known; the input is unknown.

$h(x) = 11$; $4x - 9 = 11$

Step 2

Before: $4x - 9 = 11$

Action: Apply the same inverse addition to both sides.

Operation: $4x - 9 = 11 \rightarrow 4x - 9 + (9) = 11 + (9)$

After: $4x - 9 + (9) = 11 + (9)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rclcl} 4x - 9 & = & 11 \\ + 9 & & + 9 \\ \hline 4x & \overset{=}{\cancel{-9}} & \overset{+}{\cancel{9}} & = & 20 \end{array}$$

Opposite terms sum to zero.

$$4x = 20$$

Apply + 9 to both sides

$4x - 9$ equals 11. Apply + 9 to each side. Result: $4x$ equals 20. The marked opposite terms sum to zero.

Step 3

Before: $-9+(9)$

Action: Cancel the additive inverse pair.

Operation: $-9+(9) \rightarrow -9+(9)=0; 4x+0=11+(9)$

After: $-9+(9)=0; 4x+0=11+(9)$

Why: Opposite added terms become zero. The variable term remains.

$$-9+(9)$$

$$-9+(9)=0; 4x+0=11+(9)$$

Cancel the additive inverse pair.

$-9+(9); -9+(9)=0; 4x+0=11+(9)$

Step 4

Before: $11+(9)$

Action: Calculate the right-hand side.

Operation: $11+(9) \rightarrow 20$

After: 20

Why: Finish this arithmetic before dividing.

$$11+(9)$$

$$20$$

Calculate the right-hand side.

$11+(9); 20$

Step 5

Before: $4x=20$

Action: Divide both sides by the nonzero coefficient.

Operation: $4x=20 \rightarrow (4x)/(4)=(20)/(4)$

After: $(4x)/(4)=(20)/(4)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{4x}{4} = \frac{20}{4}$$

$$\cancel{(4/4)} \times x = 20/(4)$$

$4/4=1$ because $4 \neq 0$

$$x=5$$

Divide both sides; equal nonzero factors make one

Divide $4x=20$ by 4 on each side. The coefficient quotient $4/4$ equals one, leaving $x=5$.

Step 6

Before: $(4/4)x=(20)/(4)$

Action: Reduce the coefficient quotient to one.

Operation: $(4/4)x=(20)/(4) \rightarrow 1 \times x=(20)/(4)$

After: $1 \times x=(20)/(4)$

Why: $4/4=1$, not zero; 4 is nonzero.

$$(4/4)x=(20)/(4)$$

$$1 \times x=(20)/(4)$$

Reduce the coefficient quotient to one.

$(4/4)x=(20)/(4)$; $1 \times x=(20)/(4)$

Step 7

Before: $x=(20)/(4)$

Action: Evaluate the quotient.

Operation: $x=(20)/(4) \rightarrow x=5$

After: $x=5$

Why: Keep an exact fractional value if the division is not integral.

$$x=(20)/(4)$$

$$x=5$$

Evaluate the quotient.

$x=(20)/(4)$; $x=5$

Step 8

Before: $x=5$

Action: Substitute the candidate into the original rule.

Operation: $x=5 \rightarrow h(5)=(4)(5)+(-9)$

After: $h(5)=(4)(5)+(-9)$

Why: Check the rule before applying any contextual restriction.

$$x=5$$

$$h(5)=(4)(5)+(-9)$$

Substitute the candidate into the original rule.

$x=5$; $h(5)=(4)(5)+(-9)$

Step 9

Before: $(4)(5)$

Action: Calculate the variable term.

Operation: $(4)(5) \rightarrow 20$

After: 20

Why: The product recovers the right-hand side before the constant was removed.

$$(4)(5)$$

$$20$$

Calculate the variable term.

$(4)(5)$; 20

Step 10

Before: $20+(-9)$

Action: Restore the constant and compare with the target.

Operation: $20+(-9) \rightarrow 11=11$

After: $11=11$

Why: The algebraic candidate produces the required output.

$$20+(-9)$$

$$11=11$$

Restore the constant and compare with the target.

$20+(-9); 11=11$

Step 11

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow x=5$

After: $x=5$

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$$x=5$$

Report the input with its meaning and unit.

State the requested result; $x=5$

1. The output is known; the input is unknown. $h(x)=11 \rightarrow 4x-9=11$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $4x - 9=11 \rightarrow 4x - 9+(9)=11+(9)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $-9+(9) \rightarrow -9+(9)=0$; $4x+0=11+(9)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $11+(9) \rightarrow 20$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $4x=20 \rightarrow (4x)/(4)=(20)/(4)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(4/4)x=(20)/(4) \rightarrow 1 \times x=(20)/(4)$ $4/4=1$, not zero; 4 is nonzero.
7. Evaluate the quotient. $x=(20)/(4) \rightarrow x=5$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $x=5 \rightarrow h(5)=(4)(5)+(-9)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(4)(5) \rightarrow 20$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $20+(-9) \rightarrow 11=11$ The algebraic candidate produces the required output.
11. Report the input with its meaning and unit. State the requested result $\rightarrow x=5$ The final statement answers the original question after both the algebraic and domain checks.

Answer: $x=5$

$$4x - 9 = 11.$$

Add 9 on both sides:

$$4x - 9 + 9 = 11 + 9.$$

$$4x + 0 = 20; 4x = 20.$$

Divide both sides by 4:

$$(4 \times x)/(4) = 20/(4).$$

$$(4/4) \times x = 5; 1 \times x = 5.$$

$$x = 5.$$

$$\text{Check: } 4 \times (5) - 9 = 20 - 9 = 11.$$

The original output is recovered.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Eleven is the given output.

Set $4x - 9 = 11$ and add nine on both sides.

$4x = 20$. Divide both sides by four and check.

Question 22

$h(x) = -2x + 3$. Find x when $h(x) = 9$.

Answer and working

Step 1

Before: $h(x) = 9$

Action: The output is known; the input is unknown.

Operation: $h(x) = 9 \rightarrow -2x + 3 = 9$

After: $-2x + 3 = 9$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$h(x) = 9$$

$$-2x + 3 = 9$$

The output is known; the input is unknown.

$h(x) = 9$; $-2x + 3 = 9$

Step 2

Before: $-2x + 3 = 9$

Action: Apply the same inverse addition to both sides.

Operation: $-2x + 3 = 9 \rightarrow -2x + 3 + (-3) = 9 + (-3)$

After: $-2x + 3 + (-3) = 9 + (-3)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl}
 -2x + 3 & = & 9 \\
 -3 & & -3 \\
 \hline
 -2x & \quad \quad \quad \cancel{+3} \quad \quad \quad \cancel{-3} & = 6
 \end{array}$$

Opposite terms sum to zero.

$$-2x = 6$$

Apply -3 to both sides

$-2x + 3$ equals 9. Apply -3 to each side. Result: $-2x$ equals 6. The marked opposite terms sum to zero.

Step 3

Before: $3+(-3)$

Action: Cancel the additive inverse pair.

Operation: $3+(-3) \rightarrow 3+(-3)=0$; $-2x+0=9+(-3)$

After: $3+(-3)=0$; $-2x+0=9+(-3)$

Why: Opposite added terms become zero. The variable term remains.

$$3+(-3)$$

$$3+(-3)=0; -2x+0=9+(-3)$$

Cancel the additive inverse pair.

$3+(-3)$; $3+(-3)=0$; $-2x+0=9+(-3)$

Step 4

Before: $9+(-3)$

Action: Calculate the right-hand side.

Operation: $9+(-3) \rightarrow 6$

After: 6

Why: Finish this arithmetic before dividing.

$$9+(-3)$$

$$6$$

Calculate the right-hand side.

$9+(-3)$; 6

Step 5

Before: $-2x=6$

Action: Divide both sides by the nonzero coefficient.

Operation: $-2x=6 \rightarrow (-2x)/(-2)=(6)/(-2)$

After: $(-2x)/(-2)=(6)/(-2)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{-2x}{-2} = \frac{6}{-2}$$

$$\cancel{(-2/-2)} \times x = 6/(-2)$$

$$-2/-2=1 \text{ because } -2 \neq 0$$

$$x=-3$$

Divide both sides; equal nonzero factors make one

Divide $-2x=6$ by -2 on each side. The coefficient quotient $-2/-2$ equals one, leaving $x=-3$.

Step 6

Before: $(-2/-2)x=(6)/(-2)$

Action: Reduce the coefficient quotient to one.

Operation: $(-2/-2)x=(6)/(-2) \rightarrow 1 \times x=(6)/(-2)$

After: $1 \times x=(6)/(-2)$

Why: $-2/-2=1$, not zero; -2 is nonzero.

$$(-2/-2)x=(6)/(-2)$$

$$1 \times x=(6)/(-2)$$

Reduce the coefficient quotient to one.

$(-2/-2)x=(6)/(-2)$; $1 \times x=(6)/(-2)$

Step 7

Before: $x=(6)/(-2)$

Action: Evaluate the quotient.

Operation: $x=(6)/(-2) \rightarrow x=-3$

After: $x=-3$

Why: Keep an exact fractional value if the division is not integral.

$$x=(6)/(-2)$$

$$x=-3$$

Evaluate the quotient.

$x=(6)/(-2)$; $x=-3$

Step 8

Before: $x = -3$

Action: Substitute the candidate into the original rule.

Operation: $x = -3 \rightarrow h(-3) = (-2)(-3) + (3)$

After: $h(-3) = (-2)(-3) + (3)$

Why: Check the rule before applying any contextual restriction.

$$x = -3$$

$$h(-3) = (-2)(-3) + (3)$$

Substitute the candidate into the original rule.

$x = -3$; $h(-3) = (-2)(-3) + (3)$

Step 9

Before: $(-2)(-3)$

Action: Calculate the variable term.

Operation: $(-2)(-3) \rightarrow 6$

After: 6

Why: The product recovers the right-hand side before the constant was removed.

$$(-2)(-3)$$

$$6$$

Calculate the variable term.

$(-2)(-3)$; 6

Step 10

Before: $6+(3)$

Action: Restore the constant and compare with the target.

Operation: $6+(3) \rightarrow 9=9$

After: $9=9$

Why: The algebraic candidate produces the required output.

$$6+(3)$$

$$9=9$$

Restore the constant and compare with the target.

$$6+(3); 9=9$$

Step 11

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow x=-3$

After: $x=-3$

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$$x=-3$$

Report the input with its meaning and unit.

State the requested result; $x=-3$

1. The output is known; the input is unknown. $h(x)=9 \rightarrow -2x+3=9$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $-2x + 3=9 \rightarrow -2x + 3+(-3)=9+(-3)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $3+(-3) \rightarrow 3+(-3)=0$; $-2x+0=9+(-3)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $9+(-3) \rightarrow 6$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $-2x=6 \rightarrow (-2x)/(-2)=(6)/(-2)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(-2/-2)x=(6)/(-2) \rightarrow 1 \times x=(6)/(-2)$ $-2/-2=1$, not zero; -2 is nonzero.
7. Evaluate the quotient. $x=(6)/(-2) \rightarrow x=-3$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $x=-3 \rightarrow h(-3)=(-2)(-3)+3$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(-2)(-3) \rightarrow 6$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $6+3 \rightarrow 9=9$ The algebraic candidate produces the required output.
11. Report the input with its meaning and unit. State the requested result $\rightarrow x=-3$ The final statement answers the original question after both the algebraic and domain checks.

Answer: $x=-3$

$$-2x + 3 = 9.$$

Subtract 3 on both sides:

$$-2x + 3 - 3 = 9 - 3.$$

$$-2x + 0 = 6; -2x = 6.$$

Divide both sides by -2 :

$$(-2 \times x)/(-2) = 6/(-2).$$

$$(-2/-2) \times x = -3; 1 \times x = -3.$$

$$x = -3.$$

$$\text{Check: } -2 \times (-3) + 3 = 6 + 3 = 9.$$

Dividing a positive number by a negative number gives a negative input.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Find the input producing output nine.

Set $-2x + 3 = 9$; subtract three on both sides.

$-2x = 6$. Divide both sides by -2 ; use the sign bridge if needed.

Question 23

A learner writes $f(-2) = -f(2)$ for every function. Test $f(x) = x^2 + 1$.

Answer and working

Step 1

Before: $f(-2)$

Action: Read the supplied input.

Operation: $f(-2) \rightarrow x = -2$

After: $x = -2$

Why: The number inside the brackets is the input; the required result is the output.

$$f(-2)$$

$$x = -2$$

Read the supplied input.

$f(-2)$; $x = -2$

Step 2

Before: $f(x)=x^2+1$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x^2+1 \rightarrow f(-2)=(-2)^2+1$

After: $f(-2)=(-2)^2+1$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x^2+1$$

$$f(-2)=(-2)^2+1$$

Replace each occurrence of the input variable.

$f(x)=x^2+1$; $f(-2)=(-2)^2+1$

Step 3

Before: $(-2)^2$

Action: Write the square as a product.

Operation: $(-2)^2 \rightarrow (-2) \times (-2)$

After: $(-2) \times (-2)$

Why: Both factors contain the whole input, including its sign.

$$(-2)^2$$

$$(-2) \times (-2)$$

Write the square as a product.

$(-2)^2$; $(-2) \times (-2)$

Step 4

Before: $(-2) \times (-2)$

Action: Multiply the two identical inputs.

Operation: $(-2) \times (-2) \rightarrow 4$

After: 4

Why: Two negative factors give a positive product.

$$(-2) \times (-2)$$

$$4$$

Multiply the two identical inputs.

$(-2) \times (-2); 4$

Step 5

Before: $(4) + (1)$

Action: Apply the constant term.

Operation: $(4) + (1) \rightarrow 5$

After: 5

Why: The constant is added once after the variable terms have been evaluated.

$$(4) + (1)$$

$$5$$

Apply the constant term.

$(4) + (1); 5$

Step 6

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(-2)=5$

After: $f(-2)=5$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(-2)=5$$

Use the original function name.

Label the computed output; $f(-2)=5$

Step 7

Before: $f(2)$

Action: Read the supplied input.

Operation: $f(2) \rightarrow x=2$

After: $x=2$

Why: The number inside the brackets is the input; the required result is the output.

$$f(2)$$

$$x=2$$

Read the supplied input.

$f(2); x=2$

Step 8

Before: $f(x)=x^2+1$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x^2+1 \rightarrow f(2)=(2)^2 + 1$

After: $f(2)=(2)^2 + 1$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x^2+1$$

$$f(2)=(2)^2 + 1$$

Replace each occurrence of the input variable.

$$f(x)=x^2+1; f(2)=(2)^2 + 1$$

Step 9

Before: $(2)^2$

Action: Write the square as a product.

Operation: $(2)^2 \rightarrow (2) \times (2)$

After: $(2) \times (2)$

Why: Both factors contain the whole input, including its sign.

$$(2)^2$$

$$(2) \times (2)$$

Write the square as a product.

$$(2)^2; (2) \times (2)$$

Step 10

Before: $(2) \times (2)$

Action: Multiply the two identical inputs.

Operation: $(2) \times (2) \rightarrow 4$

After: 4

Why: A square is nonnegative.

$$(2) \times (2)$$

$$4$$

Multiply the two identical inputs.

$(2) \times (2); 4$

Step 11

Before: $(4) + (1)$

Action: Apply the constant term.

Operation: $(4) + (1) \rightarrow 5$

After: 5

Why: The constant is added once after the variable terms have been evaluated.

$$(4) + (1)$$

$$5$$

Apply the constant term.

$(4) + (1); 5$

Step 12

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(2)=5$

After: $f(2)=5$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(2)=5$$

Use the original function name.

Label the computed output; $f(2)=5$

Step 13

Before: $-f(2)$

Action: Apply the minus outside the already calculated output.

Operation: $-f(2) \rightarrow -(5)=-5$

After: $-(5)=-5$

Why: An outside minus reverses the output sign; it does not change the input.

$$-f(2)$$

$$-(5)=-5$$

Apply the minus outside the already calculated output.

$-f(2); -(5)=-5$

Step 14

Before: $f(-2)=5$; $-f(2)=-5$

Action: Compare the two instructions.

Operation: $f(-2)=5$; $-f(2)=-5 \rightarrow 5 \neq -5$

After: $5 \neq -5$

Why: This single function and input are a counterexample to a claim about every function.

$$f(-2)=5; -f(2)=-5$$

$$5 \neq -5$$

Compare the two instructions.

$f(-2)=5$; $-f(2)=-5$; $5 \neq -5$

1. Read the supplied input. $f(-2) \rightarrow x=-2$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $f(x)=x^2+1 \rightarrow f(-2)=(-2)^2 + 1$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Write the square as a product. $(-2)^2 \rightarrow (-2) \times (-2)$ Both factors contain the whole input, including its sign.
4. Multiply the two identical inputs. $(-2) \times (-2) \rightarrow 4$ Two negative factors give a positive product.
5. Apply the constant term. $(4)+(1) \rightarrow 5$ The constant is added once after the variable terms have been evaluated.
6. Use the original function name. Label the computed output $\rightarrow f(-2)=5$ The requested input has now passed through every operation in the rule.
7. Read the supplied input. $f(2) \rightarrow x=2$ The number inside the brackets is the input; the required result is the output.
8. Replace each occurrence of the input variable. $f(x)=x^2+1 \rightarrow f(2)=(2)^2 + 1$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
9. Write the square as a product. $(2)^2 \rightarrow (2) \times (2)$ Both factors contain the whole input, including its sign.
10. Multiply the two identical inputs. $(2) \times (2) \rightarrow 4$ A square is nonnegative.
11. Apply the constant term. $(4)+(1) \rightarrow 5$ The constant is added once after the variable terms have been evaluated.
12. Use the original function name. Label the computed output $\rightarrow f(2)=5$ The requested input has now passed through every operation in the rule.
13. Apply the minus outside the already calculated output. $-f(2) \rightarrow -(5)=-5$ An outside minus reverses the output sign; it does not change the input.
14. Compare the two instructions. $f(-2)=5$; $-f(2)=-5 \rightarrow 5 \neq -5$ This single function and input are a counterexample to a claim about every function.

Answer: The universal claim fails: $f(-2)=5$ but $-f(2)=-5$.

$f(-2)=(-2)^2+1=4+1=5$. $f(2)=2^2+1=5$, so $-f(2)=-5$.

Five is not -5 ; the claim fails for this rule.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Compare the two claimed values separately.

Compute $f(-2)$ and $f(2)$ using $x^2 + 1$.

Both outputs are five, but $-f(2)$ is negative five. Decide whether the claim survives.

Question 24

Does the table $(2, 4), (3, 4), (4, 4)$ represent a function on these inputs?

Answer and working

Step 1

Before: Input set $[2, 3, 4]$; pairs $[(2, 4), (3, 4), (4, 4)]$

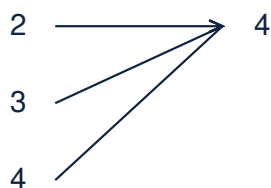
Action: Interpret each ordered pair as an association.

Operation: Input set $[2, 3, 4]$; pairs $[(2, 4), (3, 4), (4, 4)] \rightarrow$ The first coordinate is the input x ; the second is its associated output y .

After: The first coordinate is the input x ; the second is its associated output y .

Why: A relation is an association between elements of the two sets.

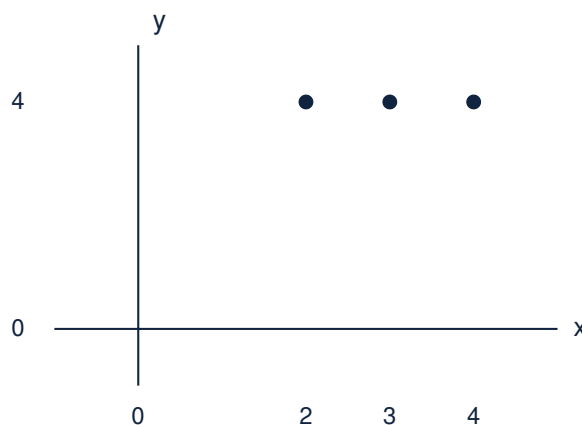
Input $\rightarrow$ output



Ordered pairs

$(2, 4)$
 $(3, 4)$
 $(4, 4)$

The same points on axes



Each arrow, pair and plotted point records the same association.

One relation in three representations

Given input set $[2, 3, 4]$, the relation contains pairs $[(2, 4), (3, 4), (4, 4)]$. The left mapping arrows, ordered-pair list and discrete point graph represent those same pairs; no connecting segments or extra inputs are implied.

Step 2

Before: Input $x=2$

Action: Collect its distinct associated outputs.

Operation: Input $x=2 \rightarrow$ Outputs [4]; count 1

After: Outputs [4]; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=2$

Outputs [4]; count 1

Collect its distinct associated outputs.

Input $x=2$; Outputs [4]; count 1

Step 3

Before: Input $x=3$

Action: Collect its distinct associated outputs.

Operation: Input $x=3 \rightarrow$ Outputs [4]; count 1

After: Outputs [4]; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=3$

Outputs [4]; count 1

Collect its distinct associated outputs.

Input $x=3$; Outputs [4]; count 1

Step 4

Before: Input $x=4$

Action: Collect its distinct associated outputs.

Operation: Input $x=4 \rightarrow$ Outputs $[4]$; count 1

After: Outputs $[4]$; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=4$

Outputs $[4]$; count 1

Collect its distinct associated outputs.

Input $x=4$; Outputs $[4]$; count 1

Step 5

Before: Apply exactly one output to every stated input

Action: State the decision and its reason.

Operation: Apply exactly one output to every stated input $\rightarrow$ Function on the stated input set; $f(2)=4$, $f(3)=4$, $f(4)=4$

After: Function on the stated input set; $f(2)=4$, $f(3)=4$, $f(4)=4$

Why: A single missing or multiple-output input breaks the function condition on the specified set.

Apply exactly one output to every stated input

Function on the stated input set; $f(2)=4$, $f(3)=4$, $f(4)=4$

State the decision and its reason.

Apply exactly one output to every stated input; Function on the stated input set; $f(2)=4$, $f(3)=4$, $f(4)=4$

1. Interpret each ordered pair as an association. Input set $[2, 3, 4]$; pairs $[(2, 4), (3, 4), (4, 4)] \rightarrow$ The first coordinate is the input x ; the second is its associated output y . A relation is an association between elements of the two sets.
2. Collect its distinct associated outputs. Input $x=2 \rightarrow$ Outputs $[4]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
3. Collect its distinct associated outputs. Input $x=3 \rightarrow$ Outputs $[4]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
4. Collect its distinct associated outputs. Input $x=4 \rightarrow$ Outputs $[4]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
5. State the decision and its reason. Apply exactly one output to every stated input $\rightarrow$ Function on the stated input set; $f(2)=4, f(3)=4, f(4)=4$ A single missing or multiple-output input breaks the function condition on the specified set.

Answer: Function on the stated input set; $f(2)=4, f(3)=4, f(4)=4$

Input $2 \rightarrow 4$, input $3 \rightarrow 4$, input $4 \rightarrow 4$.

Each input has exactly one output, so yes.

Equal outputs do not break the definition.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Check each input separately.

Group rows for inputs two, three and four.

Each has output four only. Decide whether sharing an output violates the definition.

Question 25

Does $(2, 4), (2, 7), (3, 4)$ represent a function? Name the exact conflict.

Answer and working

Step 1

Before: Input set $[2, 3]$; pairs $[(2, 4), (2, 7), (3, 4)]$

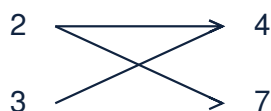
Action: Interpret each ordered pair as an association.

Operation: Input set $[2, 3]$; pairs $[(2, 4), (2, 7), (3, 4)] \rightarrow$ The first coordinate is the input x ; the second is its associated output y .

After: The first coordinate is the input x ; the second is its associated output y .

Why: A relation is an association between elements of the two sets.

Input $\rightarrow$ output



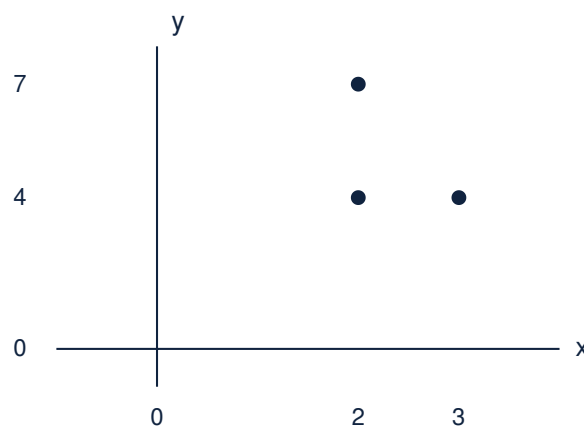
Ordered pairs

$(2, 4)$

$(2, 7)$

$(3, 4)$

The same points on axes



Each arrow, pair and plotted point records the same association.

One relation in three representations

Given input set $[2, 3]$, the relation contains pairs $[(2, 4), (2, 7), (3, 4)]$. The left mapping arrows, ordered-pair list and discrete point graph represent those same pairs; no connecting segments or extra inputs are implied.

Step 2

Before: Input $x=2$

Action: Collect its distinct associated outputs.

Operation: Input $x=2 \rightarrow$ Outputs $[4, 7]$; count 2

After: Outputs $[4, 7]$; count 2

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=2$

Outputs $[4, 7]$; count 2

Collect its distinct associated outputs.

Input $x=2$; Outputs $[4, 7]$; count 2

Step 3

Before: Input $x=3$

Action: Collect its distinct associated outputs.

Operation: Input $x=3 \rightarrow$ Outputs $[4]$; count 1

After: Outputs $[4]$; count 1

Why: Repeated identical pairs do not create a second distinct output; different inputs may share an output.

Input $x=3$

Outputs $[4]$; count 1

Collect its distinct associated outputs.

Input $x=3$; Outputs $[4]$; count 1

Step 4

Before: Apply exactly one output to every stated input

Action: State the decision and its reason.

Operation: Apply exactly one output to every stated input → Not a function on the stated input set: input 2 has 2 distinct outputs

After: Not a function on the stated input set: input 2 has 2 distinct outputs

Why: A single missing or multiple-output input breaks the function condition on the specified set.

Apply exactly one output to every stated input

Not a function on the stated input set: input 2 has 2 distinct outputs

State the decision and its reason.

Apply exactly one output to every stated input; Not a function on the stated input set: input 2 has 2 distinct; outputs

1. Interpret each ordered pair as an association. Input set $[2, 3]$; pairs $[(2, 4), (2, 7), (3, 4)]$ → The first coordinate is the input x ; the second is its associated output y . A relation is an association between elements of the two sets.
2. Collect its distinct associated outputs. Input $x=2$ → Outputs $[4, 7]$; count 2 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
3. Collect its distinct associated outputs. Input $x=3$ → Outputs $[4]$; count 1 Repeated identical pairs do not create a second distinct output; different inputs may share an output.
4. State the decision and its reason. Apply exactly one output to every stated input → Not a function on the stated input set: input 2 has 2 distinct outputs A single missing or multiple-output input breaks the function condition on the specified set.

Answer: Not a function on the stated input set: input 2 has 2 distinct outputs

Input 2 has outputs 4 and 7.

One input cannot have two different outputs, so this is not a function of the stated input.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Look for one input with different outputs.

Compare the two rows that start with two.

Those outputs are four and seven. Apply exactly one output per input.

Question 26

A printer model is $C(n)=2n+6$ dollars for n whole posters. Find the cost of 5 posters.

Answer and working

Step 1

Before: $C(5)$

Action: Read the supplied input.

Operation: $C(5) \rightarrow n=5$

After: $n=5$

Why: The number inside the brackets is the input; the required result is the output.

$C(5)$

$n=5$

Read the supplied input.

$C(5); n=5$

Step 2

Before: $C(n)=2n+6$

Action: Replace each occurrence of the input variable.

Operation: $C(n)=2n+6 \rightarrow C(5)=2(5) + 6$

After: $C(5)=2(5) + 6$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$C(n)=2n+6$$

$$C(5)=2(5) + 6$$

Replace each occurrence of the input variable.

$C(n)=2n+6$; $C(5)=2(5) + 6$

Step 3

Before: $(2) \times (5)$

Action: Multiply the coefficient and the input.

Operation: $(2) \times (5) \rightarrow 10$

After: 10

Why: Complete multiplication before addition or subtraction.

$$(2) \times (5)$$

$$10$$

Multiply the coefficient and the input.

$(2) \times (5)$; 10

Step 4

Before: $(10)+(6)$

Action: Apply the constant term.

Operation: $(10)+(6) \rightarrow 16$

After: 16

Why: The constant is added once after the variable terms have been evaluated.

$$(10)+(6)$$

$$16$$

Apply the constant term.

$(10)+(6)$; 16

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow C(5)=16$

After: $C(5)=16$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$C(5)=16$$

Use the original function name.

Label the computed output; $C(5)=16$

Step 6

Before: $C(5)=16$

Action: Interpret the output with its unit.

Operation: $C(5)=16 \rightarrow$ Five posters cost 16 dollars

After: Five posters cost 16 dollars

Why: The input is a whole poster count and the output is the order cost.

$$C(5)=16$$

Five posters cost 16 dollars

Interpret the output with its unit.

$C(5)=16$; Five posters cost 16 dollars

1. Read the supplied input. $C(5) \rightarrow n=5$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $C(n)=2n+6 \rightarrow C(5)=2(5) + 6$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(2) \times (5) \rightarrow 10$ Complete multiplication before addition or subtraction.
4. Apply the constant term. $(10)+(6) \rightarrow 16$ The constant is added once after the variable terms have been evaluated.
5. Use the original function name. Label the computed output $\rightarrow C(5)=16$ The requested input has now passed through every operation in the rule.
6. Interpret the output with its unit. $C(5)=16 \rightarrow$ Five posters cost 16 dollars The input is a whole poster count and the output is the order cost.

Answer: 16 dollars

$C(5)=2(5)+6=10+6=16$ dollars.

Five two-dollar posters plus one six-dollar setup fee gives 16.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Five posters is the input; dollars are the output.

Write $C(5) = 2(5) + 6$.

Five two-dollar posters cost ten dollars. Add the fixed fee once.

Question 27

Under $C(n)=2n+6$, how many posters give an exact 20-dollar order?

Answer and working

Step 1

Before: $C(n)=20$

Action: The output is known; the input is unknown.

Operation: $C(n)=20 \rightarrow 2n+6=20$

After: $2n+6=20$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$C(n)=20$$

$$2n+6=20$$

The output is known; the input is unknown.

$C(n)=20$; $2n+6=20$

Step 2

Before: $2n + 6 = 20$

Action: Apply the same inverse addition to both sides.

Operation: $2n + 6 = 20 \rightarrow 2n + 6 + (-6) = 20 + (-6)$

After: $2n + 6 + (-6) = 20 + (-6)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl} 2n + 6 & = & 20 \\ - 6 & & - 6 \\ \hline 2n & = & 14 \end{array}$$

Opposite terms sum to zero.

$$2n = 14$$

Apply -6 to both sides

$2n + 6$ equals 20. Apply -6 to each side. Result: $2n$ equals 14. The marked opposite terms sum to zero.

Step 3

Before: $6+(-6)$

Action: Cancel the additive inverse pair.

Operation: $6+(-6) \rightarrow 6+(-6)=0$; $2n+0=20+(-6)$

After: $6+(-6)=0$; $2n+0=20+(-6)$

Why: Opposite added terms become zero. The variable term remains.

$$6+(-6)$$

$$6+(-6)=0; 2n+0=20+(-6)$$

Cancel the additive inverse pair.

$6+(-6)$; $6+(-6)=0$; $2n+0=20+(-6)$

Step 4

Before: $20+(-6)$

Action: Calculate the right-hand side.

Operation: $20+(-6) \rightarrow 14$

After: 14

Why: Finish this arithmetic before dividing.

$$20+(-6)$$

$$14$$

Calculate the right-hand side.

$20+(-6)$; 14

Step 5

Before: $2n=14$

Action: Divide both sides by the nonzero coefficient.

Operation: $2n=14 \rightarrow (2n)/(2)=(14)/(2)$

After: $(2n)/(2)=(14)/(2)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{2n}{2} = \frac{14}{2}$$

$$\cancel{(2/2)} \times n = 14/(2)$$

$$2/2=1 \text{ because } 2 \neq 0$$

$$n=7$$

Divide both sides; equal nonzero factors make one

Divide $2n=14$ by 2 on each side. The coefficient quotient $2/2$ equals one, leaving $n=7$.

Step 6

Before: $(2/2)n=(14)/(2)$

Action: Reduce the coefficient quotient to one.

Operation: $(2/2)n=(14)/(2) \rightarrow 1 \times n=(14)/(2)$

After: $1 \times n=(14)/(2)$

Why: $2/2=1$, not zero; 2 is nonzero.

$$(2/2)n=(14)/(2)$$

$$1 \times n=(14)/(2)$$

Reduce the coefficient quotient to one.

$$(2/2)n=(14)/(2); 1 \times n=(14)/(2)$$

Step 7

Before: $n=(14)/(2)$

Action: Evaluate the quotient.

Operation: $n=(14)/(2) \rightarrow n=7$

After: $n=7$

Why: Keep an exact fractional value if the division is not integral.

$$n=(14)/(2)$$

$$n=7$$

Evaluate the quotient.

$$n=(14)/(2); n=7$$

Step 8

Before: $n=7$

Action: Substitute the candidate into the original rule.

Operation: $n=7 \rightarrow C(7)=(2)(7)+(6)$

After: $C(7)=(2)(7)+(6)$

Why: Check the rule before applying any contextual restriction.

$$n=7$$

$$C(7)=(2)(7)+(6)$$

Substitute the candidate into the original rule.

$n=7; C(7)=(2)(7)+(6)$

Step 9

Before: $(2)(7)$

Action: Calculate the variable term.

Operation: $(2)(7) \rightarrow 14$

After: 14

Why: The product recovers the right-hand side before the constant was removed.

$$(2)(7)$$

$$14$$

Calculate the variable term.

$(2)(7); 14$

Step 10

Before: $14+(6)$

Action: Restore the constant and compare with the target.

Operation: $14+(6) \rightarrow 20=20$

After: $20=20$

Why: The algebraic candidate produces the required output.

$$14+(6)$$

$$20=20$$

Restore the constant and compare with the target.

$14+(6)$; $20=20$

Step 11

Before: Candidate $n=7$

Action: Check the whole-number, nonnegative input condition.

Operation: Candidate $n=7 \rightarrow$ Permitted: a nonnegative whole number.

After: Permitted: a nonnegative whole number.

Why: A fractional number of whole items cannot be ordered.

$$\text{Candidate } n=7$$

Permitted: a nonnegative whole number.

Check the whole-number, nonnegative input condition.

Candidate $n=7$; Permitted: a nonnegative whole number.

Step 12

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow n=7$ posters

After: $n=7$ posters

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$n=7$ posters

Report the input with its meaning and unit.

State the requested result; $n=7$ posters

1. The output is known; the input is unknown. $C(n)=20 \rightarrow 2n+6=20$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $2n + 6=20 \rightarrow 2n + 6+(-6)=20+(-6)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $6+(-6) \rightarrow 6+(-6)=0$; $2n+0=20+(-6)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $20+(-6) \rightarrow 14$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $2n=14 \rightarrow (2n)/(2)=(14)/(2)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(2/2)n=(14)/(2) \rightarrow 1 \times n=(14)/(2)$ $2/2=1$, not zero; 2 is nonzero.
7. Evaluate the quotient. $n=(14)/(2) \rightarrow n=7$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $n=7 \rightarrow C(7)=(2)(7)+(6)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(2)(7) \rightarrow 14$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $14+(6) \rightarrow 20=20$ The algebraic candidate produces the required output.
11. Check the whole-number, nonnegative input condition. Candidate $n=7 \rightarrow$ Permitted: a nonnegative whole number. A fractional number of whole items cannot be ordered.
12. Report the input with its meaning and unit. State the requested result $\rightarrow n=7$ posters The final statement answers the original question after both the algebraic and domain checks.

Answer: $n=7$ posters

$$2n + 6 = 20.$$

Subtract 6 on both sides:

$$2n + 6 - 6 = 20 - 6.$$

$$2n + 0 = 14; 2n = 14.$$

Divide both sides by 2:

$$(2 \times n)/(2) = 14/(2).$$

$$(2/2) \times n = 7; 1 \times n = 7.$$

$$n = 7.$$

$$\text{Check: } 2 \times (7) + 6 = 14 + 6 = 20.$$

Seven whole posters are allowed.

The order costs exactly 20 dollars.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Twenty dollars is the output.

Set $2n + 6 = 20$ and subtract six on both sides.

$2n = 14$. Divide both sides by two; check the input is whole.

Question 28

Under the same printer model, can an exact 15-dollar order be bought?

Answer and working

Step 1

Before: $C(n)=15$

Action: The output is known; the input is unknown.

Operation: $C(n)=15 \rightarrow 2n+6=15$

After: $2n+6=15$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$C(n)=15$$

$$2n+6=15$$

The output is known; the input is unknown.

$C(n)=15$; $2n+6=15$

Step 2

Before: $2n + 6 = 15$

Action: Apply the same inverse addition to both sides.

Operation: $2n + 6 = 15 \rightarrow 2n + 6 + (-6) = 15 + (-6)$

After: $2n + 6 + (-6) = 15 + (-6)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl} 2n + 6 & = & 15 \\ - 6 & & - 6 \\ \hline 2n & = & 9 \end{array}$$

Opposite terms sum to zero.

$$2n = 9$$

Apply -6 to both sides

$2n + 6$ equals 15. Apply -6 to each side. Result: $2n$ equals 9. The marked opposite terms sum to zero.

Step 3

Before: $6+(-6)$

Action: Cancel the additive inverse pair.

Operation: $6+(-6) \rightarrow 6+(-6)=0$; $2n+0=15+(-6)$

After: $6+(-6)=0$; $2n+0=15+(-6)$

Why: Opposite added terms become zero. The variable term remains.

$$6+(-6)$$

$$6+(-6)=0; 2n+0=15+(-6)$$

Cancel the additive inverse pair.

$6+(-6)$; $6+(-6)=0$; $2n+0=15+(-6)$

Step 4

Before: $15+(-6)$

Action: Calculate the right-hand side.

Operation: $15+(-6) \rightarrow 9$

After: 9

Why: Finish this arithmetic before dividing.

$$15+(-6)$$

$$9$$

Calculate the right-hand side.

$15+(-6)$; 9

Step 5

Before: $2n=9$

Action: Divide both sides by the nonzero coefficient.

Operation: $2n=9 \rightarrow (2n)/(2)=(9)/(2)$

After: $(2n)/(2)=(9)/(2)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{2n}{2} = \frac{9}{2}$$

$$\cancel{(2/2)} \times n = 9/(2)$$

$2/2=1$ because $2 \neq 0$

$$n=9/2$$

Divide both sides; equal nonzero factors make one

Divide $2n=9$ by 2 on each side. The coefficient quotient $2/2$ equals one, leaving $n=9/2$.

Step 6

Before: $(2/2)n=(9)/(2)$

Action: Reduce the coefficient quotient to one.

Operation: $(2/2)n=(9)/(2) \rightarrow 1 \times n=(9)/(2)$

After: $1 \times n=(9)/(2)$

Why: $2/2=1$, not zero; 2 is nonzero.

$$(2/2)n=(9)/(2)$$

$$1 \times n=(9)/(2)$$

Reduce the coefficient quotient to one.

$(2/2)n=(9)/(2)$; $1 \times n=(9)/(2)$

Step 7

Before: $n=(9)/(2)$

Action: Evaluate the quotient.

Operation: $n=(9)/(2) \rightarrow n=9/2$

After: $n=9/2$

Why: Keep an exact fractional value if the division is not integral.

$$n=(9)/(2)$$

$$n=9/2$$

Evaluate the quotient.

$n=(9)/(2)$; $n=9/2$

Step 8

Before: $n=9/2$

Action: Substitute the candidate into the original rule.

Operation: $n=9/2 \rightarrow C(9/2)=(2)(9/2)+(6)$

After: $C(9/2)=(2)(9/2)+(6)$

Why: Check the rule before applying any contextual restriction.

$$n=9/2$$

$$C(9/2)=(2)(9/2)+(6)$$

Substitute the candidate into the original rule.

$n=9/2$; $C(9/2)=(2)(9/2)+(6)$

Step 9

Before: $(2)(9/2)$

Action: Calculate the variable term.

Operation: $(2)(9/2) \rightarrow 9$

After: 9

Why: The product recovers the right-hand side before the constant was removed.

$$(2)(9/2)$$

$$9$$

Calculate the variable term.

$(2)(9/2)$; 9

Step 10

Before: $9+(6)$

Action: Restore the constant and compare with the target.

Operation: $9+(6) \rightarrow 15=15$

After: $15=15$

Why: The algebraic candidate produces the required output.

$$9+(6)$$

$$15=15$$

Restore the constant and compare with the target.

$9+(6)$; $15=15$

Step 11

Before: Candidate $n=9/2$

Action: Check the whole-number, nonnegative input condition.

Operation: Candidate $n=9/2 \rightarrow$ Not permitted: this input is not a nonnegative whole number.

After: Not permitted: this input is not a nonnegative whole number.

Why: A fractional number of whole items cannot be ordered.

Candidate $n=9/2$

Not permitted: this input is not a nonnegative whole number.

Check the whole-number, nonnegative input condition.

Candidate $n=9/2$; Not permitted: this input is not a nonnegative whole number.

Step 12

Before: $C(4)=(2)(4)+(6)$

Action: Multiply the whole-item count by the per-item amount.

Operation: $C(4)=(2)(4)+(6) \rightarrow 8+(6)$

After: $8+(6)$

Why: This is a permitted neighbouring count.

$$C(4)=(2)(4)+(6)$$

$$8+(6)$$

Multiply the whole-item count by the per-item amount.

$$C(4)=(2)(4)+(6); 8+(6)$$

Step 13

Before: $8+(6)$

Action: Add the fixed amount.

Operation: $8+(6) \rightarrow C(4)=14$ dollars

After: $C(4)=14$ dollars

Why: Compare this attainable cost with the required exact cost.

$$8+(6)$$

$$C(4)=14 \text{ dollars}$$

Add the fixed amount.

$$8+(6); C(4)=14 \text{ dollars}$$

Step 14

Before: $C(5)=(2)(5)+(6)$

Action: Multiply the whole-item count by the per-item amount.

Operation: $C(5)=(2)(5)+(6) \rightarrow 10+(6)$

After: $10+(6)$

Why: This is a permitted neighbouring count.

$$C(5)=(2)(5)+(6)$$

$$10+(6)$$

Multiply the whole-item count by the per-item amount.

$C(5)=(2)(5)+(6)$; $10+(6)$

Step 15

Before: $10+(6)$

Action: Add the fixed amount.

Operation: $10+(6) \rightarrow C(5)=16$ dollars

After: $C(5)=16$ dollars

Why: Compare this attainable cost with the required exact cost.

$$10+(6)$$

$$C(5)=16 \text{ dollars}$$

Add the fixed amount.

$10+(6)$; $C(5)=16$ dollars

Step 16

Before: Both neighbouring whole counts miss the target

Action: State the contextual conclusion.

Operation: Both neighbouring whole counts miss the target → No exact 15-dollar order is available.

After: No exact 15-dollar order is available.

Why: Every additional item changes the cost by the positive fixed per-item amount; no whole count lies between the neighbours.

Both neighbouring whole counts miss the target

No exact 15-dollar order is available.

State the contextual conclusion.

Both neighbouring whole counts miss the target; No exact 15-dollar order is available.

Step 17

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result → No exact 15-dollar order is available.

After: No exact 15-dollar order is available.

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

No exact 15-dollar order is available.

Report the input with its meaning and unit.

State the requested result; No exact 15-dollar order is available.

1. The output is known; the input is unknown. $C(n)=15 \rightarrow 2n+6=15$ Set the rule equal to the known output. Do not substitute the output in place of the input.
 2. Apply the same inverse addition to both sides. $2n + 6=15 \rightarrow 2n + 6+(-6)=15+(-6)$ Equality is preserved because both sides receive the same operation.
 3. Cancel the additive inverse pair. $6+(-6) \rightarrow 6+(-6)=0$; $2n+0=15+(-6)$ Opposite added terms become zero. The variable term remains.
 4. Calculate the right-hand side. $15+(-6) \rightarrow 9$ Finish this arithmetic before dividing.
 5. Divide both sides by the nonzero coefficient. $2n=9 \rightarrow (2n)/(2)=(9)/(2)$ The same nonzero divisor is applied to both sides.
 6. Reduce the coefficient quotient to one. $(2/2)n=(9)/(2) \rightarrow 1 \times n=(9)/(2)$ $2/2=1$, not zero; 2 is nonzero.
 7. Evaluate the quotient. $n=(9)/(2) \rightarrow n=9/2$ Keep an exact fractional value if the division is not integral.
 8. Substitute the candidate into the original rule. $n=9/2 \rightarrow C(9/2)=(2)(9/2)+(6)$ Check the rule before applying any contextual restriction.
 9. Calculate the variable term. $(2)(9/2) \rightarrow 9$ The product recovers the right-hand side before the constant was removed.
 10. Restore the constant and compare with the target. $9+(6) \rightarrow 15=15$ The algebraic candidate produces the required output.
 11. Check the whole-number, nonnegative input condition. Candidate $n=9/2 \rightarrow$ Not permitted: this input is not a nonnegative whole number. A fractional number of whole items cannot be ordered.
 12. Multiply the whole-item count by the per-item amount. $C(4)=(2)(4)+(6) \rightarrow 8+(6)$ This is a permitted neighbouring count.
 13. Add the fixed amount. $8+(6) \rightarrow C(4)=14$ dollars Compare this attainable cost with the required exact cost.
 14. Multiply the whole-item count by the per-item amount. $C(5)=(2)(5)+(6) \rightarrow 10+(6)$ This is a permitted neighbouring count.
 15. Add the fixed amount. $10+(6) \rightarrow C(5)=16$ dollars Compare this attainable cost with the required exact cost.
 16. State the contextual conclusion. Both neighbouring whole counts miss the target $\rightarrow$ No exact 15-dollar order is available. Every additional item changes the cost by the positive fixed per-item amount; no whole count lies between the neighbours.
 17. Report the input with its meaning and unit. State the requested result $\rightarrow$ No exact 15-dollar order is available. The final statement answers the original question after both the algebraic and domain checks.
- Answer: No exact 15-dollar order is available.

$$2n + 6 = 15.$$

Subtract 6 on both sides:

$$2n + 6 - 6 = 15 - 6.$$

$$2n + 0 = 9; 2n = 9.$$

Divide both sides by 2:

$$(2 \times n)/(2) = 9/(2).$$

$$(2/2) \times n = 4.5; 1 \times n = 4.5.$$

$$n = 4.5.$$

$$\text{Check: } 2 \times (4.5) + 6 = 9 + 6 = 15.$$

The equation gives 4.5, but whole posters are required.

$$C(4) = 2(4) + 6 = 8 + 6 = 14 \text{ dollars.}$$

$$C(5) = 2(5) + 6 = 10 + 6 = 16 \text{ dollars.}$$

No whole input gives exactly 15 dollars.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

An algebraic input must also be a whole poster count.

Set $2n + 6 = 15$ and subtract six on both sides.

$2n = 9$. Divide both sides by two, then decide whether that exact input is allowed.

Question 29

A simplified sensor model is $V(t)=3t-2$. t is seconds, $0 \leq t \leq 4$; V is volts. Find $V(4)$ and state whether $t=6$ is permitted.

Answer and working

Step 1

Before: $V(4)$

Action: Read the supplied input.

Operation: $V(4) \rightarrow t=4$

After: $t=4$

Why: The number inside the brackets is the input; the required result is the output.

$$V(4)$$

$$t=4$$

Read the supplied input.

$V(4); t=4$

Step 2

Before: $V(t)=3t-2$

Action: Replace each occurrence of the input variable.

Operation: $V(t)=3t-2 \rightarrow V(4)=3(4) + -2$

After: $V(4)=3(4) + -2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$V(t)=3t-2$$

$$V(4)=3(4) + -2$$

Replace each occurrence of the input variable.

$V(t)=3t-2; V(4)=3(4) + -2$

Step 3

Before: $(3) \times (4)$

Action: Multiply the coefficient and the input.

Operation: $(3) \times (4) \rightarrow 12$

After: 12

Why: Complete multiplication before addition or subtraction.

$$(3) \times (4)$$

$$12$$

Multiply the coefficient and the input.

$(3) \times (4)$; 12

Step 4

Before: $(12) + (-2)$

Action: Apply the constant term.

Operation: $(12) + (-2) \rightarrow 10$

After: 10

Why: Adding a negative constant is the same as subtracting its positive magnitude.

$$(12) + (-2)$$

$$10$$

Apply the constant term.

$(12) + (-2)$; 10

Step 5

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow V(4)=10$

After: $V(4)=10$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$V(4)=10$$

Use the original function name.

Label the computed output; $V(4)=10$

Step 6

Before: $t=4$

Action: Check the input before reporting the model output.

Operation: $t=4 \rightarrow 0 \leq 4 \leq 4$; the endpoint is included

After: $0 \leq 4 \leq 4$; the endpoint is included

Why: The $\leq$ symbols include both stated endpoints.

$$t=4$$

$$0 \leq 4 \leq 4; \text{ the endpoint is included}$$

Check the input before reporting the model output.

$t=4$; $0 \leq 4 \leq 4$; the endpoint is included

Step 7

Before: $V(4)=10$

Action: Label the model output.

Operation: $V(4)=10 \rightarrow 10$ volts at 4 seconds

After: 10 volts at 4 seconds

Why: The function output is a voltage, not another time.

$$V(4)=10$$

10 volts at 4 seconds

Label the model output.

$V(4)=10$; 10 volts at 4 seconds

Step 8

Before: Proposed input $t=6$

Action: Compare it with the permitted upper bound.

Operation: Proposed input $t=6 \rightarrow 6>4$; $t=6$ is outside $0 \leq t \leq 4$

After: $6>4$; $t=6$ is outside $0 \leq t \leq 4$

Why: The supplied model is authorised only on its stated interval.

Proposed input $t=6$

$6>4$; $t=6$ is outside $0 \leq t \leq 4$

Compare it with the permitted upper bound.

Proposed input $t=6$; $6>4$; $t=6$ is outside $0 \leq t \leq 4$

Step 9

Before: The time lies outside the stated domain

Action: State the restriction.

Operation: The time lies outside the stated domain $\rightarrow t=6$ is not permitted by this model

After: $t=6$ is not permitted by this model

Why: Do not use an extrapolated number as a valid output under the given model.

The time lies outside the stated domain $t=6$ is not permitted by this model

State the restriction.

The time lies outside the stated domain; $t=6$ is not permitted by this model

1. Read the supplied input. $V(4) \rightarrow t=4$ The number inside the brackets is the input; the required result is the output.
2. Replace each occurrence of the input variable. $V(t)=3t-2 \rightarrow V(4)=3(4) + -2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
3. Multiply the coefficient and the input. $(3) \times (4) \rightarrow 12$ Complete multiplication before addition or subtraction.
4. Apply the constant term. $(12) + (-2) \rightarrow 10$ Adding a negative constant is the same as subtracting its positive magnitude.
5. Use the original function name. Label the computed output $\rightarrow V(4)=10$ The requested input has now passed through every operation in the rule.
6. Check the input before reporting the model output. $t=4 \rightarrow 0 \leq 4 \leq 4$; the endpoint is included The $\leq$ symbols include both stated endpoints.
7. Label the model output. $V(4)=10 \rightarrow 10$ volts at 4 seconds The function output is a voltage, not another time.
8. Compare it with the permitted upper bound. Proposed input $t=6 \rightarrow 6 > 4$; $t=6$ is outside $0 \leq t \leq 4$ The supplied model is authorised only on its stated interval.
9. State the restriction. The time lies outside the stated domain $\rightarrow t=6$ is not permitted by this model Do not use an extrapolated number as a valid output under the given model.

Answer: $V(4)=10$ volts; $t=6$ is not permitted.

$$V(4)=3(4)-2=12-2=10 \text{ volts.}$$

Four lies in the stated interval. $t=6$ is outside it; the supplied model does not authorise a prediction there.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

The rule is restricted to times from zero to four seconds.

Substitute four: $3(4) - 2$.

Finish $12 - 2$, attach volts, then compare six seconds with the domain.

Question 30

The same sensor reports 7 volts. Find the time and check the permitted interval.

Answer and working

Step 1

Before: $V(t)=7$

Action: The output is known; the input is unknown.

Operation: $V(t)=7 \rightarrow 3t-2=7$

After: $3t-2=7$

Why: Set the rule equal to the known output. Do not substitute the output in place of the input.

$$V(t)=7$$

$$3t-2=7$$

The output is known; the input is unknown.

$V(t)=7$; $3t-2=7$

Step 2

Before: $3t - 2 = 7$

Action: Apply the same inverse addition to both sides.

Operation: $3t - 2 = 7 \rightarrow 3t - 2 + (2) = 7 + (2)$

After: $3t - 2 + (2) = 7 + (2)$

Why: Equality is preserved because both sides receive the same operation.

$$\begin{array}{rcl}
 3t - 2 & = & 7 \\
 + 2 & & + 2 \\
 \hline
 3t & \overset{=2}{\cancel{-2}} \quad \overset{\neq 2}{\cancel{+2}} & = 9
 \end{array}$$

Opposite terms sum to zero.

$$3t = 9$$

Apply + 2 to both sides

$3t - 2$ equals 7. Apply + 2 to each side. Result: $3t$ equals 9. The marked opposite terms sum to zero.

Step 3

Before: $-2+(2)$

Action: Cancel the additive inverse pair.

Operation: $-2+(2) \rightarrow -2+(2)=0; 3t+0=7+(2)$

After: $-2+(2)=0; 3t+0=7+(2)$

Why: Opposite added terms become zero. The variable term remains.

$$-2+(2)$$

$$-2+(2)=0; 3t+0=7+(2)$$

Cancel the additive inverse pair.

$-2+(2); -2+(2)=0; 3t+0=7+(2)$

Step 4

Before: $7+(2)$

Action: Calculate the right-hand side.

Operation: $7+(2) \rightarrow 9$

After: 9

Why: Finish this arithmetic before dividing.

$$7+(2)$$

$$9$$

Calculate the right-hand side.

$7+(2); 9$

Step 5

Before: $3t=9$

Action: Divide both sides by the nonzero coefficient.

Operation: $3t=9 \rightarrow (3t)/(3)=(9)/(3)$

After: $(3t)/(3)=(9)/(3)$

Why: The same nonzero divisor is applied to both sides.

$$\frac{3t}{3} = \frac{9}{3}$$

$$\cancel{(3/3)} \times t = 9/(3)$$

$3/3=1$ because $3 \neq 0$

$$t=3$$

Divide both sides; equal nonzero factors make one

Divide $3t=9$ by 3 on each side. The coefficient quotient $3/3$ equals one, leaving $t=3$.

Step 6

Before: $(3/3)t=(9)/(3)$

Action: Reduce the coefficient quotient to one.

Operation: $(3/3)t=(9)/(3) \rightarrow 1 \times t=(9)/(3)$

After: $1 \times t=(9)/(3)$

Why: $3/3=1$, not zero; 3 is nonzero.

$$(3/3)t=(9)/(3)$$

$$1 \times t=(9)/(3)$$

Reduce the coefficient quotient to one.

$(3/3)t=(9)/(3); 1 \times t=(9)/(3)$

Step 7

Before: $t=(9)/(3)$

Action: Evaluate the quotient.

Operation: $t=(9)/(3) \rightarrow t=3$

After: $t=3$

Why: Keep an exact fractional value if the division is not integral.

$$t=(9)/(3)$$

$$t=3$$

Evaluate the quotient.

$t=(9)/(3); t=3$

Step 8

Before: $t=3$

Action: Substitute the candidate into the original rule.

Operation: $t=3 \rightarrow V(3)=(3)(3)+(-2)$

After: $V(3)=(3)(3)+(-2)$

Why: Check the rule before applying any contextual restriction.

$$t=3$$

$$V(3)=(3)(3)+(-2)$$

Substitute the candidate into the original rule.

$t=3; V(3)=(3)(3)+(-2)$

Step 9

Before: $(3)(3)$

Action: Calculate the variable term.

Operation: $(3)(3) \rightarrow 9$

After: 9

Why: The product recovers the right-hand side before the constant was removed.

$$(3)(3)$$

$$9$$

Calculate the variable term.

$(3)(3); 9$

Step 10

Before: $9+(-2)$

Action: Restore the constant and compare with the target.

Operation: $9+(-2) \rightarrow 7=7$

After: $7=7$

Why: The algebraic candidate produces the required output.

$$9+(-2)$$

$$7=7$$

Restore the constant and compare with the target.

$9+(-2); 7=7$

Step 11

Before: $t=3$

Action: Check the model's stated input interval.

Operation: $t=3 \rightarrow 0 \leq 3 \leq 4$

After: $0 \leq 3 \leq 4$

Why: The time satisfies both bounds and is therefore permitted by this model.

$$t=3$$

$$0 \leq 3 \leq 4$$

Check the model's stated input interval.

$t=3; 0 \leq 3 \leq 4$

Step 12

Before: State the requested result

Action: Report the input with its meaning and unit.

Operation: State the requested result $\rightarrow t=3$ seconds

After: $t=3$ seconds

Why: The final statement answers the original question after both the algebraic and domain checks.

State the requested result

$t=3$ seconds

Report the input with its meaning and unit.

State the requested result; $t=3$ seconds

1. The output is known; the input is unknown. $V(t)=7 \rightarrow 3t-2=7$ Set the rule equal to the known output. Do not substitute the output in place of the input.
2. Apply the same inverse addition to both sides. $3t - 2=7 \rightarrow 3t - 2+(2)=7+(2)$ Equality is preserved because both sides receive the same operation.
3. Cancel the additive inverse pair. $-2+(2) \rightarrow -2+(2)=0$; $3t+0=7+(2)$ Opposite added terms become zero. The variable term remains.
4. Calculate the right-hand side. $7+(2) \rightarrow 9$ Finish this arithmetic before dividing.
5. Divide both sides by the nonzero coefficient. $3t=9 \rightarrow (3t)/(3)=(9)/(3)$ The same nonzero divisor is applied to both sides.
6. Reduce the coefficient quotient to one. $(3/3)t=(9)/(3) \rightarrow 1 \times t=(9)/(3)$ $3/3=1$, not zero; 3 is nonzero.
7. Evaluate the quotient. $t=(9)/(3) \rightarrow t=3$ Keep an exact fractional value if the division is not integral.
8. Substitute the candidate into the original rule. $t=3 \rightarrow V(3)=(3)(3)+(-2)$ Check the rule before applying any contextual restriction.
9. Calculate the variable term. $(3)(3) \rightarrow 9$ The product recovers the right-hand side before the constant was removed.
10. Restore the constant and compare with the target. $9+(-2) \rightarrow 7=7$ The algebraic candidate produces the required output.
11. Check the model's stated input interval. $t=3 \rightarrow 0 \leq 3 \leq 4$ The time satisfies both bounds and is therefore permitted by this model.
12. Report the input with its meaning and unit. State the requested result $\rightarrow t=3$ seconds The final statement answers the original question after both the algebraic and domain checks.

Answer: $t=3$ seconds

$$3t - 2 = 7.$$

Add 2 on both sides:

$$3t - 2 + 2 = 7 + 2.$$

$$3t + 0 = 9; 3t = 9.$$

Divide both sides by 3:

$$(3 \times t)/(3) = 9/(3).$$

$$(3/3) \times t = 3; 1 \times t = 3.$$

$$t = 3.$$

$$\text{Check: } 3 \times (3) - 2 = 9 - 2 = 7.$$

The time is 3 seconds.

$$\text{Check the domain: } 0 \leq 3 \leq 4.$$

The voltage is 7 volts.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Seven volts is an output, so solve for time.

Set $3t - 2 = 7$ and add two to both sides.

$3t = 9$. Divide both sides by three and check the time lies in the domain.

Question 31

A table shows only $f(0) = 2$ and $f(1) = 3$. Can you determine $f(2)$ without a rule? Explain with two possible rules.

Answer and working

Step 1

Before: Candidate rule $f(x)=x+2$

Action: Test all supplied rows before using the new input.

Operation: Candidate rule $f(x)=x+2 \rightarrow$ Check inputs 0 and 1, then calculate input 2

After: Check inputs 0 and 1, then calculate input 2

Why: A candidate must match both given values; otherwise it is irrelevant.

Candidate rule $f(x)=x+2$

Check inputs 0 and 1, then calculate input 2

Test all supplied rows before using the new input.

Candidate rule $f(x)=x+2$; Check inputs 0 and 1, then calculate input 2

Step 2

Before: $f(0)$

Action: Read the supplied input.

Operation: $f(0) \rightarrow x=0$

After: $x=0$

Why: The number inside the brackets is the input; the required result is the output.

$$f(0)$$

$$x=0$$

Read the supplied input.

$f(0); x=0$

Step 3

Before: $f(x)=x+2$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x+2 \rightarrow f(0)=(0) + 2$

After: $f(0)=(0) + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x+2$$

$$f(0)=(0) + 2$$

Replace each occurrence of the input variable.

$f(x)=x+2; f(0)=(0) + 2$

Step 4

Before: $(1) \times (0)$

Action: Multiply the coefficient and the input.

Operation: $(1) \times (0) \rightarrow 0$

Why: Complete multiplication before addition or subtraction.

$$(1) \times (0)$$

$$0$$

Multiply the coefficient and the input.

$(1) \times (0); 0$

Step 5

Before: $(0) + (2)$

Action: Apply the constant term.

Operation: $(0) + (2) \rightarrow 2$

After: 2

Why: The constant is added once after the variable terms have been evaluated.

$$(0) + (2)$$

$$2$$

Apply the constant term.

$(0) + (2); 2$

Step 6

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(0)=2$

After: $f(0)=2$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(0)=2$$

Use the original function name.

Label the computed output; $f(0)=2$

Step 7

Before: $f(1)$

Action: Read the supplied input.

Operation: $f(1) \rightarrow x=1$

After: $x=1$

Why: The number inside the brackets is the input; the required result is the output.

$$f(1)$$

$$x=1$$

Read the supplied input.

$f(1); x=1$

Step 8

Before: $f(x)=x+2$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x+2 \rightarrow f(1)=(1) + 2$

After: $f(1)=(1) + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x+2$$

$$f(1)=(1) + 2$$

Replace each occurrence of the input variable.

$f(x)=x+2$; $f(1)=(1) + 2$

Step 9

Before: $(1) \times (1)$

Action: Multiply the coefficient and the input.

Operation: $(1) \times (1) \rightarrow 1$

After: 1

Why: Complete multiplication before addition or subtraction.

$$(1) \times (1)$$

$$1$$

Multiply the coefficient and the input.

$(1) \times (1)$; 1

Step 10

Before: $(1)+(2)$

Action: Apply the constant term.

Operation: $(1)+(2) \rightarrow 3$

After: 3

Why: The constant is added once after the variable terms have been evaluated.

$$(1)+(2)$$

$$3$$

Apply the constant term.

$(1)+(2); 3$

Step 11

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(1)=3$

After: $f(1)=3$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(1)=3$$

Use the original function name.

Label the computed output; $f(1)=3$

Step 12

Before: $f(2)$

Action: Read the supplied input.

Operation: $f(2) \rightarrow x=2$

After: $x=2$

Why: The number inside the brackets is the input; the required result is the output.

$$f(2)$$

$$x=2$$

Read the supplied input.

$f(2); x=2$

Step 13

Before: $f(x)=x+2$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x+2 \rightarrow f(2)=(2) + 2$

After: $f(2)=(2) + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x+2$$

$$f(2)=(2) + 2$$

Replace each occurrence of the input variable.

$f(x)=x+2; f(2)=(2) + 2$

Step 14

Before: $(1) \times (2)$

Action: Multiply the coefficient and the input.

Operation: $(1) \times (2) \rightarrow 2$

After: 2

Why: Complete multiplication before addition or subtraction.

$$(1) \times (2)$$

$$2$$

Multiply the coefficient and the input.

$(1) \times (2); 2$

Step 15

Before: $(2) + (2)$

Action: Apply the constant term.

Operation: $(2) + (2) \rightarrow 4$

After: 4

Why: The constant is added once after the variable terms have been evaluated.

$$(2) + (2)$$

$$4$$

Apply the constant term.

$(2) + (2); 4$

Step 16

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(2)=4$

After: $f(2)=4$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(2)=4$$

Use the original function name.

Label the computed output; $f(2)=4$

Step 17

Before: Candidate rule $g(x)=x^2+2$

Action: Test all supplied rows before using the new input.

Operation: Candidate rule $g(x)=x^2+2 \rightarrow$ Check inputs 0 and 1, then calculate input 2

After: Check inputs 0 and 1, then calculate input 2

Why: A candidate must match both given values; otherwise it is irrelevant.

Candidate rule $g(x)=x^2+2$

Check inputs 0 and 1, then calculate input 2

Test all supplied rows before using the new input.

Candidate rule $g(x)=x^2+2$; Check inputs 0 and 1, then calculate input 2

Step 18

Before: $g(0)$

Action: Read the supplied input.

Operation: $g(0) \rightarrow x=0$

After: $x=0$

Why: The number inside the brackets is the input; the required result is the output.

$$g(0)$$

$$x=0$$

Read the supplied input.

$g(0); x=0$

Step 19

Before: $g(x)=x^2+2$

Action: Replace each occurrence of the input variable.

Operation: $g(x)=x^2+2 \rightarrow g(0)=(0)^2 + 2$

After: $g(0)=(0)^2 + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$g(x)=x^2+2$$

$$g(0)=(0)^2 + 2$$

Replace each occurrence of the input variable.

$g(x)=x^2+2; g(0)=(0)^2 + 2$

Step 20

Before: $(0)^2$

Action: Write the square as a product.

Operation: $(0)^2 \rightarrow (0) \times (0)$

After: $(0) \times (0)$

Why: Both factors contain the whole input, including its sign.

$$(0)^2$$

$$(0) \times (0)$$

Write the square as a product.

$(0)^2$; $(0) \times (0)$

Step 21

Before: $(0) \times (0)$

Action: Multiply the two identical inputs.

Operation: $(0) \times (0) \rightarrow 0$

Why: A square is nonnegative.

$$(0) \times (0)$$

$$0$$

Multiply the two identical inputs.

$(0) \times (0)$; 0

Step 22

Before: $(0)+(2)$

Action: Apply the constant term.

Operation: $(0)+(2) \rightarrow 2$

After: 2

Why: The constant is added once after the variable terms have been evaluated.

$$(0)+(2)$$

$$2$$

Apply the constant term.

$(0)+(2); 2$

Step 23

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow g(0)=2$

After: $g(0)=2$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$g(0)=2$$

Use the original function name.

Label the computed output; $g(0)=2$

Step 24

Before: $g(1)$

Action: Read the supplied input.

Operation: $g(1) \rightarrow x=1$

After: $x=1$

Why: The number inside the brackets is the input; the required result is the output.

$$g(1)$$

$$x=1$$

Read the supplied input.

$g(1); x=1$

Step 25

Before: $g(x)=x^2+2$

Action: Replace each occurrence of the input variable.

Operation: $g(x)=x^2+2 \rightarrow g(1)=(1)^2 + 2$

After: $g(1)=(1)^2 + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$g(x)=x^2+2$$

$$g(1)=(1)^2 + 2$$

Replace each occurrence of the input variable.

$g(x)=x^2+2; g(1)=(1)^2 + 2$

Step 26

Before: $(1)^2$

Action: Write the square as a product.

Operation: $(1)^2 \rightarrow (1) \times (1)$

After: $(1) \times (1)$

Why: Both factors contain the whole input, including its sign.

$$(1)^2$$

$$(1) \times (1)$$

Write the square as a product.

$(1)^2$; $(1) \times (1)$

Step 27

Before: $(1) \times (1)$

Action: Multiply the two identical inputs.

Operation: $(1) \times (1) \rightarrow 1$

After: 1

Why: A square is nonnegative.

$$(1) \times (1)$$

$$1$$

Multiply the two identical inputs.

$(1) \times (1)$; 1

Step 28

Before: $(1)+(2)$

Action: Apply the constant term.

Operation: $(1)+(2) \rightarrow 3$

After: 3

Why: The constant is added once after the variable terms have been evaluated.

$$(1)+(2)$$

$$3$$

Apply the constant term.

$(1)+(2); 3$

Step 29

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow g(1)=3$

After: $g(1)=3$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$g(1)=3$$

Use the original function name.

Label the computed output; $g(1)=3$

Step 30

Before: $g(2)$

Action: Read the supplied input.

Operation: $g(2) \rightarrow x=2$

After: $x=2$

Why: The number inside the brackets is the input; the required result is the output.

$$g(2)$$

$$x=2$$

Read the supplied input.

$g(2); x=2$

Step 31

Before: $g(x)=x^2+2$

Action: Replace each occurrence of the input variable.

Operation: $g(x)=x^2+2 \rightarrow g(2)=(2)^2 + 2$

After: $g(2)=(2)^2 + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$g(x)=x^2+2$$

$$g(2)=(2)^2 + 2$$

Replace each occurrence of the input variable.

$g(x)=x^2+2; g(2)=(2)^2 + 2$

Step 32

Before: $(2)^2$

Action: Write the square as a product.

Operation: $(2)^2 \rightarrow (2) \times (2)$

After: $(2) \times (2)$

Why: Both factors contain the whole input, including its sign.

$$(2)^2$$

$$(2) \times (2)$$

Write the square as a product.

$(2)^2$; $(2) \times (2)$

Step 33

Before: $(2) \times (2)$

Action: Multiply the two identical inputs.

Operation: $(2) \times (2) \rightarrow 4$

After: 4

Why: A square is nonnegative.

$$(2) \times (2)$$

$$4$$

Multiply the two identical inputs.

$(2) \times (2)$; 4

Step 34

Before: $(4)+(2)$

Action: Apply the constant term.

Operation: $(4)+(2) \rightarrow 6$

After: 6

Why: The constant is added once after the variable terms have been evaluated.

$$(4)+(2)$$

$$6$$

Apply the constant term.

$(4)+(2)$; 6

Step 35

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow g(2)=6$

After: $g(2)=6$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$g(2)=6$$

Use the original function name.

Label the computed output; $g(2)=6$

Step 36

Before: Both rules match output 2 at input 0 and output 3 at input 1

Action: Compare their predictions at input 2.

Operation: Both rules match output 2 at input 0 and output 3 at input 1 → First rule gives 4; second rule gives 6

After: First rule gives 4; second rule gives 6

Why: The two specified rows do not distinguish these rules.

Both rules match output 2 at input 0 and output 3 at input 1

First rule gives 4; second rule gives 6

Compare their predictions at input 2.

Both rules match output 2 at input 0 and output 3 at input 1; First rule gives 4; second rule gives 6

Step 37

Before: Different valid rules give different next outputs

Action: State what the table determines.

Operation: Different valid rules give different next outputs → $f(2)$ cannot be determined uniquely without more information

After: $f(2)$ cannot be determined uniquely without more information

Why: The two possible rules are examples of compatible choices, not two simultaneous definitions of the same known function.

Different valid rules give different next outputs

$f(2)$ cannot be determined uniquely without more information

State what the table determines.

Different valid rules give different next outputs; $f(2)$ cannot be determined uniquely without more information

1. Test all supplied rows before using the new input. Candidate rule $f(x)=x+2$ → Check inputs 0 and 1, then calculate input 2 A candidate must match both given values; otherwise it is irrelevant.
2. Read the supplied input. $f(0) \rightarrow x=0$ The number inside the brackets is the input; the required result is the output.
3. Replace each occurrence of the input variable. $f(x)=x+2 \rightarrow f(0)=(0) + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
4. Multiply the coefficient and the input. $(1) \times (0) \rightarrow 0$ Complete multiplication before addition or subtraction.
5. Apply the constant term. $(0)+(2) \rightarrow 2$ The constant is added once after the variable terms have been

evaluated.

6. Use the original function name. Label the computed output $\rightarrow f(0)=2$ The requested input has now passed through every operation in the rule.

7. Read the supplied input. $f(1) \rightarrow x=1$ The number inside the brackets is the input; the required result is the output.

8. Replace each occurrence of the input variable. $f(x)=x+2 \rightarrow f(1)=(1) + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

9. Multiply the coefficient and the input. $(1) \times (1) \rightarrow 1$ Complete multiplication before addition or subtraction.

10. Apply the constant term. $(1)+(2) \rightarrow 3$ The constant is added once after the variable terms have been evaluated.

11. Use the original function name. Label the computed output $\rightarrow f(1)=3$ The requested input has now passed through every operation in the rule.

12. Read the supplied input. $f(2) \rightarrow x=2$ The number inside the brackets is the input; the required result is the output.

13. Replace each occurrence of the input variable. $f(x)=x+2 \rightarrow f(2)=(2) + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

14. Multiply the coefficient and the input. $(1) \times (2) \rightarrow 2$ Complete multiplication before addition or subtraction.

15. Apply the constant term. $(2)+(2) \rightarrow 4$ The constant is added once after the variable terms have been evaluated.

16. Use the original function name. Label the computed output $\rightarrow f(2)=4$ The requested input has now passed through every operation in the rule.

17. Test all supplied rows before using the new input. Candidate rule $g(x)=x^2+2 \rightarrow$ Check inputs 0 and 1, then calculate input 2 A candidate must match both given values; otherwise it is irrelevant.

18. Read the supplied input. $g(0) \rightarrow x=0$ The number inside the brackets is the input; the required result is the output.

19. Replace each occurrence of the input variable. $g(x)=x^2+2 \rightarrow g(0)=(0)^2 + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

20. Write the square as a product. $(0)^2 \rightarrow (0) \times (0)$ Both factors contain the whole input, including its sign.

21. Multiply the two identical inputs. $(0) \times (0) \rightarrow 0$ A square is nonnegative.

22. Apply the constant term. $(0)+(2) \rightarrow 2$ The constant is added once after the variable terms have been evaluated.

23. Use the original function name. Label the computed output $\rightarrow g(0)=2$ The requested input has now passed through every operation in the rule.

24. Read the supplied input. $g(1) \rightarrow x=1$ The number inside the brackets is the input; the required result is the output.

25. Replace each occurrence of the input variable. $g(x)=x^2+2 \rightarrow g(1)=(1)^2 + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

26. Write the square as a product. $(1)^2 \rightarrow (1) \times (1)$ Both factors contain the whole input, including its sign.

27. Multiply the two identical inputs. $(1) \times (1) \rightarrow 1$ A square is nonnegative.

28. Apply the constant term. $(1)+(2) \rightarrow 3$ The constant is added once after the variable terms have been evaluated.

29. Use the original function name. Label the computed output $\rightarrow g(1)=3$ The requested input has now passed through every operation in the rule.

30. Read the supplied input. $g(2) \rightarrow x=2$ The number inside the brackets is the input; the required result is the

output.

31. Replace each occurrence of the input variable. $g(x)=x^2+2 \rightarrow g(2)=(2)^2 + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

32. Write the square as a product. $(2)^2 \rightarrow (2) \times (2)$ Both factors contain the whole input, including its sign.

33. Multiply the two identical inputs. $(2) \times (2) \rightarrow 4$ A square is nonnegative.

34. Apply the constant term. $(4)+(2) \rightarrow 6$ The constant is added once after the variable terms have been evaluated.

35. Use the original function name. Label the computed output $\rightarrow g(2)=6$ The requested input has now passed through every operation in the rule.

36. Compare their predictions at input 2. Both rules match output 2 at input 0 and output 3 at input 1 $\rightarrow$ First rule gives 4; second rule gives 6 The two specified rows do not distinguish these rules.

37. State what the table determines. Different valid rules give different next outputs $\rightarrow f(2)$ cannot be determined uniquely without more information The two possible rules are examples of compatible choices, not two simultaneous definitions of the same known function.

Answer: No unique $f(2)$: $x+2$ gives 4 whereas x^2+2 gives 6; both match the two supplied rows.

No.

Test $f(x) = x + 2$: $f(0) = 0 + 2 = 2$ and $f(1) = 1 + 2 = 3$, then $f(2) = 2 + 2 = 4$.

Test $g(x) = x^2 + 2$: $g(0) = 0^2 + 2 = 0 + 2 = 2$ and $g(1) = 1^2 + 2 = 1 + 2 = 3$, but $g(2) = 2^2 + 2 = 4 + 2 = 6$.

Both fit the known rows but disagree at input two.

Ask for the rule or more conditions.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

Only two input-output pairs are given.

Compare the candidate rules $x + 2$ and $x^2 + 2$ at inputs zero and one.

Both fit. Their values at two are $2 + 2$ and $2^2 + 2$. Decide whether the missing output is determined.

Question 32

A machine squares its input, then adds two. Write its rule and compare the outputs for -3 and 3 . Explain whether equal outputs break the function rule.

Answer and working

Step 1

Before: Square the input, then add two

Action: Write the rule in the stated order.

Operation: Square the input, then add two $\rightarrow f(x)=x^2+2$

After: $f(x)=x^2+2$

Why: The entire input is squared before the constant is added.

Square the input, then add two

$$f(x)=x^2+2$$

Write the rule in the stated order.

Square the input, then add two; $f(x)=x^2+2$

Step 2

Before: $f(-3)$

Action: Read the supplied input.

Operation: $f(-3) \rightarrow x=-3$

After: $x=-3$

Why: The number inside the brackets is the input; the required result is the output.

$$f(-3)$$

$$x=-3$$

Read the supplied input.

$f(-3); x=-3$

Step 3

Before: $f(x)=x^2+2$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x^2+2 \rightarrow f(-3)=(-3)^2 + 2$

After: $f(-3)=(-3)^2 + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x^2+2$$

$$f(-3)=(-3)^2 + 2$$

Replace each occurrence of the input variable.

$f(x)=x^2+2; f(-3)=(-3)^2 + 2$

Step 4

Before: $(-3)^2$

Action: Write the square as a product.

Operation: $(-3)^2 \rightarrow (-3) \times (-3)$

After: $(-3) \times (-3)$

Why: Both factors contain the whole input, including its sign.

$$(-3)^2$$

$$(-3) \times (-3)$$

Write the square as a product.

$(-3)^2$; $(-3) \times (-3)$

Step 5

Before: $(-3) \times (-3)$

Action: Multiply the two identical inputs.

Operation: $(-3) \times (-3) \rightarrow 9$

After: 9

Why: Two negative factors give a positive product.

$$(-3) \times (-3)$$

$$9$$

Multiply the two identical inputs.

$(-3) \times (-3)$; 9

Step 6

Before: $(9)+(2)$

Action: Apply the constant term.

Operation: $(9)+(2) \rightarrow 11$

After: 11

Why: The constant is added once after the variable terms have been evaluated.

$$(9)+(2)$$

$$11$$

Apply the constant term.

$(9)+(2)$; 11

Step 7

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(-3)=11$

After: $f(-3)=11$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(-3)=11$$

Use the original function name.

Label the computed output; $f(-3)=11$

Step 8

Before: $f(3)$

Action: Read the supplied input.

Operation: $f(3) \rightarrow x=3$

After: $x=3$

Why: The number inside the brackets is the input; the required result is the output.

$$f(3)$$

$$x=3$$

Read the supplied input.

$f(3); x=3$

Step 9

Before: $f(x)=x^2+2$

Action: Replace each occurrence of the input variable.

Operation: $f(x)=x^2+2 \rightarrow f(3)=(3)^2 + 2$

After: $f(3)=(3)^2 + 2$

Why: Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.

$$f(x)=x^2+2$$

$$f(3)=(3)^2 + 2$$

Replace each occurrence of the input variable.

$f(x)=x^2+2; f(3)=(3)^2 + 2$

Step 10

Before: $(3)^2$

Action: Write the square as a product.

Operation: $(3)^2 \rightarrow (3) \times (3)$

After: $(3) \times (3)$

Why: Both factors contain the whole input, including its sign.

$$(3)^2$$

$$(3) \times (3)$$

Write the square as a product.

$(3)^2$; $(3) \times (3)$

Step 11

Before: $(3) \times (3)$

Action: Multiply the two identical inputs.

Operation: $(3) \times (3) \rightarrow 9$

After: 9

Why: A square is nonnegative.

$$(3) \times (3)$$

$$9$$

Multiply the two identical inputs.

$(3) \times (3)$; 9

Step 12

Before: $(9)+(2)$

Action: Apply the constant term.

Operation: $(9)+(2) \rightarrow 11$

After: 11

Why: The constant is added once after the variable terms have been evaluated.

$$(9)+(2)$$

$$11$$

Apply the constant term.

$(9)+(2)$; 11

Step 13

Before: Label the computed output

Action: Use the original function name.

Operation: Label the computed output $\rightarrow f(3)=11$

After: $f(3)=11$

Why: The requested input has now passed through every operation in the rule.

Label the computed output

$$f(3)=11$$

Use the original function name.

Label the computed output; $f(3)=11$

Step 14

Before: $f(-3)=11$ and $f(3)=11$

Action: Compare the two outputs.

Operation: $f(-3)=11$ and $f(3)=11 \rightarrow$ Different inputs -3 and 3 share output 11

After: Different inputs -3 and 3 share output 11

Why: Squaring removes the difference between these opposite input signs.

$$f(-3)=11 \text{ and } f(3)=11$$

Different inputs -3 and 3 share output 11

Compare the two outputs.

$f(-3)=11$ and $f(3)=11$; Different inputs -3 and 3 share output 11

Step 15

Before: Check the function condition input by input

Action: Count outputs for each input.

Operation: Check the function condition input by input $\rightarrow$ Each input still has exactly one output

After: Each input still has exactly one output

Why: The function condition prohibits two outputs for one input, not two inputs sharing one output.

Check the function condition input by input

Each input still has exactly one output

Count outputs for each input.

Check the function condition input by input; Each input still has exactly one output

1. Write the rule in the stated order. Square the input, then add two $\rightarrow f(x)=x^2+2$ The entire input is squared before the constant is added.
 2. Read the supplied input. $f(-3) \rightarrow x=-3$ The number inside the brackets is the input; the required result is the output.
 3. Replace each occurrence of the input variable. $f(x)=x^2+2 \rightarrow f(-3)=(-3)^2 + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
 4. Write the square as a product. $(-3)^2 \rightarrow (-3) \times (-3)$ Both factors contain the whole input, including its sign.
 5. Multiply the two identical inputs. $(-3) \times (-3) \rightarrow 9$ Two negative factors give a positive product.
 6. Apply the constant term. $(9)+(2) \rightarrow 11$ The constant is added once after the variable terms have been evaluated.
 7. Use the original function name. Label the computed output $\rightarrow f(-3)=11$ The requested input has now passed through every operation in the rule.
 8. Read the supplied input. $f(3) \rightarrow x=3$ The number inside the brackets is the input; the required result is the output.
 9. Replace each occurrence of the input variable. $f(x)=x^2+2 \rightarrow f(3)=(3)^2 + 2$ Keep a negative or fractional input inside a complete bracket; the constant remains unchanged.
 10. Write the square as a product. $(3)^2 \rightarrow (3) \times (3)$ Both factors contain the whole input, including its sign.
 11. Multiply the two identical inputs. $(3) \times (3) \rightarrow 9$ A square is nonnegative.
 12. Apply the constant term. $(9)+(2) \rightarrow 11$ The constant is added once after the variable terms have been evaluated.
 13. Use the original function name. Label the computed output $\rightarrow f(3)=11$ The requested input has now passed through every operation in the rule.
 14. Compare the two outputs. $f(-3)=11$ and $f(3)=11 \rightarrow$ Different inputs -3 and 3 share output 11 Squaring removes the difference between these opposite input signs.
 15. Count outputs for each input. Check the function condition input by input $\rightarrow$ Each input still has exactly one output The function condition prohibits two outputs for one input, not two inputs sharing one output.
- Answer: $f(x)=x^2+2$; $f(-3)=f(3)=11$. Shared outputs do not violate the function rule.

$$f(x)=x^2+2. f(-3)=(-3)^2+2=9+2=11. f(3)=3^2+2=9+2=11.$$

Each input has one output, so equal outputs are allowed.

What to look for

- Uses the method appropriate to the question.
- Shows all necessary arithmetic or reasoning.
- Checks the original rule, units and permitted inputs.

Hints to offer

The order is square first, then add two.

Write the rule $x^2 + 2$; substitute -3 and 3 separately.

Both squares are nine. Add two to each; explain the one-output rule for each input.

Teaching notes

Allow completion across as many sessions as required. No lesson timer or assumed completion time.

Ask which value is input before any calculation. If the learner guesses, return to the input-rule-output diagram.

Accept valid alternative working with the same checks. A correct number with an invalid cancellation does not demonstrate the method.

After independent accuracy, revisit with fresh values. Optional fluency practice must not remove the reasoning or demand a fixed speed.

Come back — fresh retrieval

Retrieval 1

For $f(x)=5x-4$, find $f(-2)$.

$f(-2)=5(-2)-4=-10-4=-14$. The bracket contains the input; multiply before subtracting.

Retrieval 2

A table lists (4, 7), (5, 7), (6, 8). Is it a function?

Yes. Each input has one output. Inputs four and five sharing output seven is allowed.

Retrieval 3

Whole notebooks cost $C(n)=3n+5$ dollars. Is an exact \$16 order possible?

$3n + 5 = 16$. Subtract five on both sides: $3n + 5 - 5 = 16 - 5$; $3n + 0 = 11$; $3n = 11$. Divide both sides by three: $(3 \times n)/3 = 11/3$; $(3/3) \times n = 11/3$; $1 \times n = 11/3$; $n = 11/3$. This is not whole, so no. $C(3) = 3(3) + 5 = 9 + 5 = 14$ dollars; $C(4) = 3(4) + 5 = 12 + 5 = 17$ dollars.

Retrieval 4

Only $f(0)=3$ and $f(1)=4$ are known. Explain why $f(2)$ need not be five.

The rule $f(x)=x+3$ gives $0+3=3$ and $1+3=4$, then $2+3=5$. The rule $g(x)=x^2+3$ gives $0^2+3=3$ and $1^2+3=4$, then $2^2+3=7$. Both fit the known data, so the missing output is undetermined.

Apply your understanding

Construct two different functions that both send 0 to 1 and 1 to 2 but send 2 to different outputs. Explain why two rows do not determine a rule.

Answer and working

$f(x)=x+1$: $f(0)=1$, $f(1)=2$, $f(2)=3$. $g(x)=x^2+1$: $g(0)=1$, $g(1)=2$, $g(2)=5$. Both fit the given rows but disagree at 2, so additional information is required.

Check common errors

$f(3)$ means multiply f by three.

The bracket specifies the input; use the function rule to calculate its output.

Different inputs cannot share an output.

The definition restricts outputs for each one input. x^2 assigns the same output to 2 and -2 .

Cancel a term inside a sum.

Divide every term, or factor the whole numerator first. Test a simple input to expose an invalid cancellation.

Lesson review

- I can identify input, rule and output in a function.
- I can substitute a positive or negative input and calculate in order.
- I can find an input for a stated linear output using equal operations.
- I can interpret and check a function model in its stated context.